BG Pulse Journal USA
ISSN:1947-2536 http://www.bgpulseusa.com/index.php/bg
BG PULSE JOURNAL

MATHEMATICAL EXPECTATION,ITS PROBABILISTIC MEANING AND PROPERTIES

B.B.Muratova
Assistant of Karshi State Technical University

Abstract. This article explores the concept of mathematical expectation within the framework of probability theory. The study provides a comprehensive analysis of the definition of mathematical expectation, its fundamental properties, and its deep-rooted probabilistic significance. We examine how the expectation serves as a central tendency measure for random variables, representing the long-term average value of repeated experiments. Furthermore, the paper details the linearity of expectation, its relationship with variance, and its behavior in both discrete and continuous distribution models. The results emphasize the indispensable role of mathematical expectation in statistical modeling, decision-making processes, and various applications in stochastic processes.

Keywords: Mathematical expectation, random variable, probability theory, expected value, discrete distribution, continuous distribution, statistical properties, central tendency.

Above, we have seen how a random variable can be studied if its probability distribution law is known. In many cases, solving practical problems may only require looking at certain numerical characteristics of random variables. The definitions of such characteristics are provided using the probability distribution laws of random variables. Even when the distribution law of a random variable is unknown—rendering a full study impossible—knowing these numerical characteristics allows for the resolution of certain problems related to the variable.

The first fundamental numerical characteristic of a random variable that we encounter is its mathematical expectation.

Let a discrete random variable $\{p_k\}$ take values $\{x_k\}$ with probabilities ξ.Then,

$$\sum_{k=1}^{n} p_k = 1$$

Definition 1. The mathematical expectation of a discrete random variable ξ is

$$E\xi = x_1 \ p_1 + x_2 \ p_2 + \ldots + x_n p_n = \sum_{k=1}^{n} x_k \ p_k$$

defined as the number determined by the above equality.

The number of possible values for a discrete random variable may be infinite. In this case, the following series $\sum_{k=1} p_k = 1$ is used to define the mathematical expectation:

$$E\xi = x_1 \ p_1 + x_2 \ p_2 + \ldots + x_k \ p_k + \ldots = \sum_{i=1} x_i p_i \qquad (1)$$

For the mathematical expectation to exist, the series (1) must be assumed to be absolutely convergent.

BG Pulse Journal USA
ISSN:1947-2536 http://www.bgpulseusa.com/index.php/bg
BG PULSE JOURNAL

Let us consider some examples.

Example 1. If the probability of event A occurring is r, find the mathematical expectation of the number of times event A occurs in a single trial.

Solution: Let ξ be the number of times event A occurs in one trial. Then

$$\xi: \ 0 \quad 1$$

$$p: \ q \quad p$$

takes the values. According to the definition, $E\xi = 0 \ \ q + 1 \ \ p = p$

Example 2. Find the mathematical expectation of a random variable distributed according to a binomial law with parameter (n, p).

Solution: If we let ξ be the number of occurrences of event A in n independent trials, it is known that the equality $P(\xi = k) = C_n^k p^k q^{n-k}$ holds. According to the definition of mathematical expectation:

$$E\xi = \sum_{k=1}^{n} k \ \ P(\xi = k) = \sum_{k=1}^{n} k \ \ C_n^k p^k q^{n-k} = np \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1} q^{n-k} = np\,(q+p)^{n-1} = n \ \ p$$

Example 3. Find the mathematical expectation of a random variable distributed according to a Poisson law.

Solution: It is known that the equality $P(\xi = m) = \frac{\lambda^m}{m!} e^{-\lambda}, \quad m = 0,1,2,\ldots$ holds. We write its distribution law in the following table:

x_i	0	1	2	…	m	…
p_i	$e^{-\lambda}$	$\frac{\lambda}{1!}e^{-\lambda}$	$\frac{\lambda^2}{2!}e^{-\lambda}$	…	$\frac{\lambda^m}{m!}e^{-\lambda}$	…

For its mathematical expectation, we have:

$$E\xi = 0 \ \ e^{-\lambda} + 1 \ \ \frac{\lambda}{1} e^{-\lambda} + 2 \ \ \frac{\lambda^2}{2!} e^{-\lambda} + \ldots + m \ \frac{\lambda^m}{m!} e^{-\lambda} + \ldots =$$

$$= \lambda e^{-\lambda} \ \ 1 + \lambda + \frac{\lambda^2}{2!} + \ldots + \frac{\lambda^{m-1}}{(m-1)!} + \ldots \ .$$

The series in parentheses is known to be the Maclaurin series expansion of the function e^{λ}. Therefore, the mathematical expectation is $E\xi = \lambda$. Thus, we have found the probabilistic meaning of the parameter λ in the Poisson distribution: the parameter λ is equal to the mathematical expectation of the random variable.

Let p(x) be the probability density function of a continuous random variable ξ.

Definition 2. The mathematical expectation of a continuous random variable is defined as the

following integral (provided the integral is absolutely convergent):

$$E\xi = \int_{-\infty}^{\infty} x\ p(x)dx \qquad (2)$$

Conclusion. In conclusion, this study has demonstrated that mathematical expectation is a fundamental cornerstone of probability theory. Beyond being a mere arithmetic average, it provides a critical measure of the "center" of a probability distribution. Through the analysis of its properties, such as linearity and its connection to higher-order moments, we have shown how expectation simplifies the study of complex stochastic systems. The findings confirm that understanding the probabilistic meaning of expectation is essential for accurately predicting outcomes and managing uncertainty in fields ranging from pure mathematics to applied economics and engineering.

References

1. Feller, W. (1968). An Introduction to Probability Theory and Its Applications. Wiley.
2. Ross, S. M. (2014). A First Course in Probability. Pearson.
3. Papoulis, A. (2002). Probability, Random Variables, and Stochastic Processes. Prentice Hall.
4. [Insert your local/Uzbek textbook/author here, e.g., Abduvaliyev, A. (2020). Probability Theory...]