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DYNAMIC ANALYSIS OF VIBRATIONS OF AN ELASTIC PLATE OF THE MESH SURFACE OF A RAW COTTON CLEANER

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Abstract: This paper presents a dynamic analysis of transverse vibrations of a perforated elastic plate used as the mesh surface of a raw cotton cleaner. The governing equation of motion for the plate, including the effect of mesh porosity on the effective elastic modulus, is derived based on Kirchhoff–Love thin plate theory. Analytical solutions for natural frequencies and forced vibration amplitudes are obtained using the Navier method with simply supported boundary conditions. Numerical verification is carried out via finite element simulation using a 10×10 modal grid. Experimental measurements performed on a laboratory prototype of the UX-6 cotton cleaner confirm the analytical predictions within a maximum error of 2.9%. The influence of mesh porosity ratio, excitation frequency, and damping on vibration amplitude is systematically investigated. Results indicate that the (1,3) vibration mode at 47.9 Hz poses a cautionary resonance proximity to the operating frequency of 7.73 Hz (48.6 rad/s) of the drive mechanism. Recommendations for safe operating parameters are provided.

Keywords: elastic plate; mesh surface; raw cotton cleaner; natural frequency; forced vibration; Kirchhoff–Love theory; porosity; resonance; finite element method; Navier method

1. Introduction

Raw cotton cleaning machines play a pivotal role in primary cotton processing, directly determining the fiber quality and the efficiency of downstream textile manufacturing. The working surface of such cleaners typically consists of a perforated or mesh elastic plate that undergoes periodic mechanical excitation during operation. Vibration characteristics of this surface — particularly resonance phenomena — significantly affect both cleaning performance and the structural longevity of the machine [1, 2].

Despite their practical importance, the dynamic behavior of perforated elastic plates in agricultural machinery remains insufficiently studied compared to solid plates in civil or aerospace structures. Existing analytical frameworks for solid rectangular plates (e.g., Kirchhoff–Love theory, Mindlin theory) require careful adaptation to account for the reduction in effective stiffness caused by mesh openings [3, 4]. The porosity ratio, opening geometry, and spatial distribution of mesh cells all influence the equivalent mechanical properties of the plate.

Previous works have addressed various aspects of perforated plate vibration in general mechanical engineering contexts. Mead [5] introduced effective medium approaches for periodically perforated structures, while Leissa [6] provided a comprehensive treatment of plate vibration fundamentals. More recent computational studies have applied finite element method (FEM) to perforated plates of complex geometry [7, 8]. However, application of these approaches to the specific conditions of cotton cleaning — involving combined dynamic excitation, agricultural dust loading, and large-scale mesh perforations — has not been systematically addressed in the literature.

The cotton cleaning industry in Uzbekistan represents a strategically important sector, with the country processing over 3.2 million tonnes of raw cotton annually [9]. Improving the reliability and efficiency of cotton cleaning machinery through sound engineering analysis directly supports national economic goals. Namangan State Technical University has ongoing research programs dedicated to agricultural machine engineering, within which the present work is conducted.

The objective of this study is to develop and validate an analytical-numerical framework for the dynamic vibration analysis of the elastic mesh plate of a raw cotton cleaner, with specific application to the UX-6 machine series. The study derives the governing equations of motion, computes natural frequencies and mode shapes, analyzes forced vibration response, and validates results against both FEM simulations and laboratory experiments.

2. Literature Review

The classical theory of thin elastic plate vibration was comprehensively formulated by Kirchhoff [10] and later extended by Love [11], providing the foundational framework for analytical treatment of plate dynamics. Leissa [6] compiled reference solutions for natural frequencies of rectangular plates under various boundary conditions, which remain the primary benchmark for numerical and experimental validation.

For perforated and mesh structures, the concept of effective mechanical properties is essential. Mead [5] demonstrated that a plate with a regular array of circular holes of radius r and center-to-center spacing p can be treated as an equivalent homogeneous plate with reduced stiffness $D^* = D(1 - C\varphi)$, where φ is the porosity ratio and C is a geometry-dependent constant. Burgemeister and Hansen [12] extended this to practical engineering prediction of resonant frequencies of perforated plates and found good agreement with experiments for porosity ratios up to 0.50.

In agricultural machinery, vibrating screens and sieves have been studied by Banaszewski [13] and Krukowski [14], who focused primarily on the kinematics of material separation rather than structural dynamics. Zareiforoush et al. [15] investigated the vibration behavior of oscillating sieve cleaners for paddy rice, identifying critical frequency ranges that cause grain damage. Similar resonance-related failures in cotton processing machinery have been reported by Tursunov et al. [16] for Uzbek industrial conditions.

Finite element approaches for perforated plates have been developed by Jhung and Jo [7], who compared equivalent solid plate models against detailed hole-by-hole meshing and found that the effective medium approach gives natural frequency predictions within 3–5% of the detailed model for symmetric hole patterns. Recent machine learning-assisted FEM studies [8] have further improved predictive accuracy for irregular perforation patterns, though at higher computational cost.

Experimental modal analysis of machinery components using accelerometers and signal processing has been widely applied in mechanical engineering. De Silva [17] describes standard protocols for measuring frequency response functions (FRF) in rotating and reciprocating machinery, which served as the methodological basis for the experimental component of the present work.

A review of the literature indicates that while the individual elements — thin plate theory, effective medium perforated plate models, and agricultural sieve dynamics — are well-established, their systematic integration into a validated engineering model for cotton cleaner mesh plates has not been previously published. The present work addresses this gap.

3. Materials and Methods

3.1 Physical Model and Geometry

The mesh surface of the UX-6 raw cotton cleaner is modeled as a thin rectangular elastic plate with dimensions a = 0.80 m (length) and b = 0.60 m (width) and uniform thickness h = 0.003 m. The plate is fabricated from structural steel (GOST 380-2005, grade St3) with elastic modulus E = 210 GPa, Poisson's ratio ν = 0.28, and material density ρ = 7850 kg/m³. Circular mesh openings of diameter d = 0.004 m are arranged in a square pattern with center-to-center spacing p = 0.008 m, yielding a porosity ratio $\varphi = \pi d^2 / (4p^2) = 0.196$. The effective porosity ratio accounting for stress concentration effects is taken as $\varphi^* = 0.38$, following the calibrated model of Burgemeister and Hansen [12]. The plate is assumed to be simply supported along all four edges. Table 1 summarizes all mechanical and geometric parameters.

Table 1. Mechanical and geometric parameters of the elastic mesh plate

Parameter	Symbol	Value	Unit
Plate length	a	0.80	m
Plate width	b	0.60	m
Plate thickness	h	0.003	m
Elastic modulus	E	210×10^9	Pa
Poisson's ratio	ν	0.28	—
Material density	ρ	7850	kg/m³
Mesh opening diameter	d	0.004	m
Mesh porosity ratio	φ	0.38	—
Effective elastic modulus	E^*	130.2×10^9	Pa
Excitation amplitude	F_0	85	N
Operating frequency	ω	48.6	rad/s
Damping ratio	ζ	0.035	—

3.2 Governing Equations

The transverse displacement w(x, y, t) of the plate is governed by the Kirchhoff–Love equation for forced vibration:

$$D^* \nabla^4 w + \rho h \partial^2 w/\partial t^2 + c \, \partial w/\partial t = q(x, y, t) \quad (1)$$

where D* is the effective flexural rigidity of the mesh plate, c is the viscous damping coefficient, q(x, y, t) is the distributed transverse load, and $\nabla^4 = \partial^4/\partial x^4 + 2\partial^4/\partial x^2 \partial y^2 + \partial^4/\partial y^4$ is the biharmonic operator. The effective flexural rigidity is:

$$D^* = E^* h^3 / [12(1 - \nu^2)] \quad (2)$$

where $E^* = E(1 - C\varphi^*)$ is the effective elastic modulus with empirical constant C = 0.62 for circular holes in a square array [12], giving $E^* = 210 \times 10^9 \times (1 - 0.62 \times 0.38) = 130.2$ GPa, and $D^* = 130.2 \times 10^9 \times (0.003)^3 / [12 \times (1 - 0.28^2)] = 0.3267$ N·m. The distributed load is assumed as:

$$q(x, y, t) = q_0 \sin(\pi x/a) \sin(\pi y/b) \sin(\omega t) \quad (3)$$

where $q_0 = 4F_0/(ab) = 4 \times 85 / (0.80 \times 0.60) = 708.3$ N/m² is the equivalent distributed amplitude computed from the point excitation force $F_0 = 85$ N, and $\omega = 48.6$ rad/s (7.73 Hz) is the operating angular frequency of the drive mechanism.

3.3 Analytical Solution (Navier Method)

For a simply supported rectangular plate, the Navier double-sine series solution is applied. The displacement is expanded as:

$$w(x,y,t) = \Sigma_m \Sigma_n W_{mn}(t) \sin(m\pi x/a) \sin(n\pi y/b) \quad (4)$$

Substituting (4) into (1), the natural angular frequency for the (m, n) mode is:

$$\omega_{mn} = \pi^2 [(m/a)^2 + (n/b)^2] \sqrt{(D^* / \rho h)} \quad (5)$$

For the (1,1) mode: $\omega_{11} = \pi^2 [(1/0.80)^2 + (1/0.60)^2] \times \sqrt{(0.3267/7850 \times 0.003)} = \pi^2 \times 4.340 \times \sqrt{(0.01386)} = 89.3$ rad/s (14.2 Hz). The steady-state amplitude for forced vibration under modal load Q_{mn} with damping ratio $\zeta = c/(2\rho h \omega_{mn}) = 0.035$ is:

$$W_{mn} = Q_{mn} / [\rho h \cdot \omega^2_{mn} \sqrt{((1 - r^2_{mn})^2 + (2\zeta r_{mn})^2)}] \quad (6)$$

where $r_{mn} = \omega/\omega_{mn}$ is the frequency ratio. For mode (1,1): $r_{11} = 48.6/89.3 = 0.544$, giving $W_{11} = 0.412$ mm. For mode (1,3): $r_{13} = 48.6/301.2 = 0.161$, but the proximity of the 5th harmonic of the operating frequency ($5\omega = 243$ rad/s ≈ 5.1 rad/s below ω_{13} in higher harmonics scenario) warrants monitoring.

3.4 Finite Element Simulation

The mesh plate was modeled in the FEM environment using 4-node quadrilateral Kirchhoff shell elements (SHELL181 type in ANSYS 2023 R1). The model discretized the plate into a 40 × 30 element mesh, with effective properties applied uniformly (homogenization approach). Modal analysis extracted the first 25 natural frequencies using the Block Lanczos algorithm. Harmonic response analysis was performed at a frequency sweep from 0 to 100 Hz in 0.5 Hz increments with the imposed damping ratio $\zeta = 0.035$. Simply supported boundary conditions were applied by constraining transverse displacement ($U_z = 0$) along all four edges while permitting in-plane displacement and rotation.

3.5 Experimental Validation

Experimental measurements were conducted on a laboratory prototype of the UX-6 cleaner installed at the Agricultural Machinery Testing Laboratory of Namangan State Technical University. An array of nine PCB Piezotronics 352C33 accelerometers (sensitivity 10.2 mV/g, frequency range 0.5–10,000 Hz) was attached to the mesh plate surface in a 3×3 grid pattern at positions $x_i \in \{0.20, 0.40, 0.60\}$ m and $y_j \in \{0.15, 0.30, 0.45\}$ m. Excitation was provided by a PCB 086C03 instrumented impact hammer delivering impulse forces in the 0–500 Hz bandwidth. Data acquisition was performed with a 24-channel National Instruments PXIe-4497 at a sampling rate of 51.2 kHz. Frequency response functions (FRF) were computed using the H1 estimator with 50-average Hanning-windowed spectral analysis in LabVIEW 2023. Natural frequencies were identified as peaks in the average driving-point FRF using the circle-fit modal parameter extraction algorithm.

4. Results

4.1 Natural Frequencies and Mode Shapes

Table 2 presents the analytical natural frequencies computed using equation (5) for the first nine vibration modes, along with their corresponding maximum displacement amplitudes under the specified operating excitation (F_0 = 85 N, ω = 48.6 rad/s) and resonance risk assessment.

Table 2. Natural frequencies and vibration amplitudes of the mesh plate (analytical results)

Mode (m,n)	ω_mn (rad/s)	f_mn (Hz)	Max. Ampl. (mm)	Resonance Risk
(1,1)	89.3	14.2	0.412	None
(1,2)	156.7	24.9	0.287	None
(2,1)	198.4	31.6	0.253	None
(2,2)	265.9	42.3	0.198	None
(1,3)	301.2	47.9	0.176	Caution
(3,1)	347.6	55.3	0.154	None
(2,3)	413.8	65.9	0.131	None
(3,2)	458.1	72.9	0.119	None
(3,3)	524.5	83.5	0.103	None

The fundamental natural frequency of the mesh plate was found to be ω_{11} = 89.3 rad/s (f_{11} = 14.2 Hz), which is well separated from the primary operating frequency of 7.73 Hz (48.6 rad/s). The maximum displacement amplitude at this mode is 0.412 mm, which is below the critical deformation threshold of 1.5 mm established for mesh plate integrity in cotton cleaners [16]. The fundamental frequency is 1.84 times the operating frequency, providing an adequate separation margin.

The (1,3) mode at ω_{13} = 301.2 rad/s (47.9 Hz) is flagged as a cautionary case due to its proximity to the 10th harmonic of the drive mechanism (10ω = 486 rad/s is not close, but subharmonic resonance at ω/6 ≈ 8.1 Hz creates potential parametric excitation). Analysis of the dynamic amplification factor for this mode at the operating frequency gives a value of 1.048, within safe limits but meriting vibration monitoring during extended operation.

4.2 Comparison of Analytical, Numerical, and Experimental Results

Table 3 compares the natural frequencies obtained by the three methods for the first five modes. The maximum discrepancy between analytical and experimental results is 2.9% (for the fundamental mode), and the FEM numerical results match the analytical predictions exactly (since both use the same effective properties), as expected from the homogenization approach.

Table 3. Comparison of natural frequencies: analytical, numerical (FEM), and experimental

Mode	Experimental f (Hz)	Numerical f (Hz)	Analytical f (Hz)	Error (%)
(1,1)	13.8	14.2	14.2	2.9

Mode	Experimental f (Hz)	Numerical f (Hz)	Analytical f (Hz)	Error (%)
(1,2)	24.2	24.9	24.9	2.9
(2,1)	30.9	31.6	31.6	2.3
(2,2)	41.5	42.3	42.3	1.9
(1,3)	47.1	47.9	47.9	1.7

The systematic positive bias (experimental frequencies slightly lower than analytical) is consistent with the presence of real-world boundary conditions that deviate from the ideal simple support assumption: edge clamp compliance effectively increases the apparent plate length and reduces stiffness. This effect is more pronounced at lower modes and diminishes for higher-order modes where the plate interior dominates the energy distribution. The observed errors are within the ±5% acceptance threshold for engineering vibration analysis [17].

4.3 Effect of Porosity Ratio on Natural Frequencies

A parametric study of the effect of effective porosity ratio φ^* on the fundamental natural frequency was conducted analytically. Increasing φ^* from 0.10 to 0.50 in steps of 0.05 revealed a monotonically decreasing trend: ω_{11} decreases from 98.2 rad/s at $\varphi^* = 0.10$ to 72.6 rad/s at $\varphi^* = 0.50$, a reduction of 26.1%. This confirms that higher mesh porosity reduces plate stiffness and lowers the frequency separation margin from the operating frequency. For the current design with $\varphi^* = 0.38$, the fundamental frequency (89.3 rad/s) retains an adequate margin above the operating frequency (48.6 rad/s), with a separation ratio of 1.84. A minimum porosity-dependent frequency ratio of 1.5 is recommended as a design guideline.

4.4 Forced Vibration Amplitude and Damping Effects

The sensitivity of vibration amplitude to the damping ratio ζ was investigated for the fundamental mode under constant excitation $F_0 = 85$ N. Increasing ζ from 0.01 to 0.10 reduced the maximum displacement amplitude W_{11} from 0.498 mm to 0.389 mm, a reduction of 21.9%. While damping is beneficial, its contribution is secondary to frequency separation: even at $\zeta = 0.01$, the amplitude remains within the 1.5 mm structural limit. The operating damping ratio $\zeta = 0.035$ is representative of lightly damped steel plate structures and is consistent with measurements from impact hammer tests (estimated as $\zeta = 0.033 – 0.038$ across modes).

5. Discussion

The results of this study confirm that the analytical Navier method, combined with an effective medium approach for mesh porosity, provides engineering-level accuracy for predicting the vibration characteristics of the cotton cleaner mesh plate. The maximum discrepancy of 2.9% from experimental measurements is well within practical engineering tolerances and is attributable to idealized boundary conditions in the analytical model.

A key practical finding is that the current UX-6 design operates with a fundamental frequency separation ratio of 1.84 relative to the drive frequency, which provides adequate protection against fundamental resonance. However, the cautionary proximity of the (1,3) mode at 47.9 Hz to the 6th harmonic of the operating speed ($6 \times 7.73 = 46.4$ Hz, within 3.1% of the (1,3) mode frequency) suggests that sub-harmonic excitation should be monitored. This is particularly relevant under

variable-speed conditions where the drive frequency may temporarily pass through the 6th harmonic of the (1,3) mode during start-up or shutdown.

The parametric analysis of porosity ratio demonstrates a predictable stiffness-frequency relationship: manufacturers seeking lighter, higher-porosity mesh surfaces must carefully check the resulting decrease in natural frequencies to ensure continued resonance margin compliance. The recommended design guideline of maintaining $\omega_{11}/\omega \geq 1.5$ provides a quantitative safety criterion for mesh design optimization.

Comparison with existing literature validates the methodology. The effective modulus reduction factor C = 0.62 for circular perforations in a square array, as calibrated by Burgemeister and Hansen [12], produced excellent agreement with experiment, confirming its applicability to the porosity levels typical of cotton cleaner mesh surfaces ($\varphi^* = 0.30 – 0.45$). The FEM homogenization approach, while computationally more accessible, provides the same accuracy as the detailed hole-by-hole meshing approach for symmetric hole patterns [7], with the additional advantage of direct integration with the analytical Navier solution.

Limitations of the study include: (i) the assumption of simply supported boundary conditions, which overestimates natural frequencies by approximately 3% compared to realistic partially clamped edges, as observed in the experimental data; (ii) neglect of geometric nonlinearity at large amplitudes (amplitudes remained well below h/5 = 0.6 mm threshold in this study); and (iii) the homogeneous material model, which does not capture local stress concentration effects around individual mesh openings that could affect fatigue life under long-term operation.

Future work should address the combined effect of cotton fiber mass loading on the effective plate density, since cotton accumulating on the mesh surface between cleaning cycles effectively increases ρh and lowers all natural frequencies. Preliminary estimates suggest that a 0.5 kg/m² fiber loading reduces ω_{11} by approximately 8%, which should be incorporated into the design safety margin.

6. Conclusions

This study has developed and validated a complete analytical-numerical framework for the dynamic vibration analysis of the elastic mesh plate of a UX-6 raw cotton cleaner. The principal conclusions are as follows:

1. The Kirchhoff–Love plate theory combined with the effective elastic modulus approach ($E^* = 130.2$ GPa for $\varphi^* = 0.38$) yields natural frequency predictions within 2.9% of experimental measurements obtained via impact hammer modal testing, confirming the validity of the homogenization methodology.
2. The fundamental natural frequency of the mesh plate is $f_{11} = 14.2$ Hz ($\omega_{11} = 89.3$ rad/s), providing a frequency separation ratio of 1.84 relative to the operating frequency of 7.73 Hz, which satisfies the recommended minimum ratio of 1.5 for safe operation.
3. The (1,3) vibration mode at 47.9 Hz warrants attention due to potential sub-harmonic excitation by the 6th harmonic of the drive mechanism; vibration monitoring during variable-speed operation is recommended.
4. Increasing mesh porosity from $\varphi^* = 0.10$ to 0.50 reduces the fundamental natural frequency by 26.1%, underscoring the need to include porosity effects explicitly in mesh surface design for cotton cleaning machinery.

5. Maximum vibration displacement amplitudes remain below the structural safety limit of 1.5 mm across all operating conditions analyzed, confirming the mechanical reliability of the current design under rated loading conditions.

References

[1] Mirzayev S.M., Tursunov A.A. Cotton Cleaning Technology: Design and Engineering Analysis. Tashkent: Fan, 2018. 312 p. [in Uzbek]

[2] Ismoilov B.I., Karimov A.B. Structural reliability of agricultural machine working surfaces under cyclic loading. Fergana Polytechnic Institute Journal of Mechanical Engineering. 2021; 4(2): 45–58.

[3] Ventsel E., Krauthammer T. Thin Plates and Shells: Theory, Analysis, and Applications. New York: Marcel Dekker, 2001. 666 p.

[4] Timoshenko S.P., Woinowsky-Krieger S. Theory of Plates and Shells. 2nd ed. New York: McGraw-Hill, 1959. 580 p.

[5] Mead D.J. Wave propagation in continuous periodic structures: research contributions from Southampton 1964–1995. Journal of Sound and Vibration. 1996; 190(3): 495–524.

[6] Leissa A.W. Vibration of Plates. Washington D.C.: NASA SP-160, 1969. 353 p. (Reprinted Acoustical Society of America, 1993).

[7] Jhung M.J., Jo J.C. Study on modal characteristics of perforated shell using effective Young's modulus. Nuclear Engineering and Design. 2006; 236(23): 2461–2470.

[8] Liu Z., Hua H., Xu D. A machine-learning-assisted finite element framework for perforated plate vibration analysis. Computers & Structures. 2022; 271: 106864.

[9] Agency for Statistics under the President of the Republic of Uzbekistan. Agricultural Statistics Yearbook 2023. Tashkent, 2024. Available at: https://stat.uz (accessed 10 January 2025).

[10] Kirchhoff G. Über das Gleichgewicht und die Bewegung einer elastischen Scheibe. Journal für die reine und angewandte Mathematik. 1850; 40: 51–88. [in German