BG Pulse Journal USA
ISSN:1947-2536 http://www.bgpulseusa.com/index.php/bg
BG PULSE JOURNAL

Geometric Analysis of Architectural Domes and Vaults Using Descriptive Geometry Methods

Maxmudova Saboxat
Lecturer, Department of Architecture,
Samarkand State Technical University (SDTU).
doi:https://doi.org/10.5281/zenodo.23689173

Abstract — Domes and vaults represent foundational arcuated structural systems in architectural history, spanning vast interior volumes through complex double-curvature surfaces and stereotomic masonry assemblies. However, teaching the geometric construction of barrel vaults, groin vaults, cloister vaults, and spherical domes over square bays (via squinches and pendentives) poses severe pedagogical challenges due to the high cognitive load of 3D mental rotation. This study evaluates the effectiveness of Gaspard Monge's double orthographic projection method integrated with physical stereotomic models and 3D digital verification (N = 120 architecture students). Over a 14-week empirical intervention, spatial visualization growth was quantitatively assessed using the Revised Purdue Spatial Visualization Test (PSVT:R) and stereotomic error-identification rubrics. Results demonstrate that the hybrid descriptive geometry approach improved spatial visualization scores by +61.6% compared to a 2D drafting control group (+12.4%) and elevated geometric error detection accuracy to 89.6%

Keywords: Descriptive Geometry, Double Orthographic Projection, Domes and Vaults, Stereotomy, Pendentives, Groin Vaults, PSVT:R, Spatial Ability, Architectural Education.

1. Introduction

Arcuated architectural systems—encompassing barrel vaults, groin vaults, cloister vaults, and hemispherical domes—have historically empowered master builders to span expansive spatial enclosures using stone and brick masonry in pure compression. The precise geometric definition and stonecutting layout of these curved enclosures rely fundamentally on the science of stereotomy: the art and technique of cutting three-dimensional solids to assemble complex vaulting surfaces (Calvo-López, 2020). From Roman aqueducts and Gothic cathedral rib vaults to Renaissance domes, the spatial definition of intersecting curved surfaces required sophisticated geometric tools long before the advent of computer-aided design (Casonato, 2016).

In 1799, French mathematician and physicist Gaspard Monge formalized these centuries-old stonecutting practices into Géométrie descriptive (Descriptive Geometry). Monge's double orthographic projection system established a mathematically rigorous method for projecting 3D spatial geometry onto two mutually perpendicular 2D planes: the Horizontal plane (H, representing plans) and the Vertical plane (V, representing front elevations), separated by a ground reference line (XY). This system enabled architects and engineers to systematically construct true surface shapes (vraie grandeur), true dihedral joint angles, and complex 3D surface intersection curves (such as the elliptical groins resulting from intersecting semi-cylindrical barrels) entirely on paper using straightedge and compass (Lawrence et al., 2019).

Despite its historical and mathematical significance, modern architectural education often struggles to teach descriptive geometry effectively. First-year architecture students frequently experience severe cognitive overload when attempting to mentally rotate 3D vaulted surfaces and

BG Pulse Journal USA
ISSN:1947-2536 http://www.bgpulseusa.com/index.php/bg
BG PULSE JOURNAL

translate orthographic projections into spatial structures. Novices routinely struggle to visualize how a square plan bay transitions into a circular dome drum via squinches or spherical pendentives.

2. Methodology

2.1 Research Design and Student Sample

A 14-week quasi-experimental empirical study was conducted across two academic semesters with N = 120 undergraduate architecture students enrolled in a compulsory Architectural Graphics and Descriptive Geometry course. Participants were randomly divided into three equal cohort groups (n = 40 each):

1. 1. Control Group (Group A - 2D Drafting): Taught vault geometry exclusively through traditional 2D paper drafting, static orthographic diagrams, and manual textbook exercises.

2. 2. Experimental Group B (Stereotomic DG): Taught using Gaspard Monge's formal double orthographic projection rules combined with hands-on physical foam-block carving of stone vault joints.

3. 3. Experimental Group C (Hybrid DG + 3D): Taught using Monge's double orthographic projections synchronized with parametric 3D surface modeling (Rhinoceros/Grasshopper) and 1:1 scale Virtual Reality (VR) spatial audit.

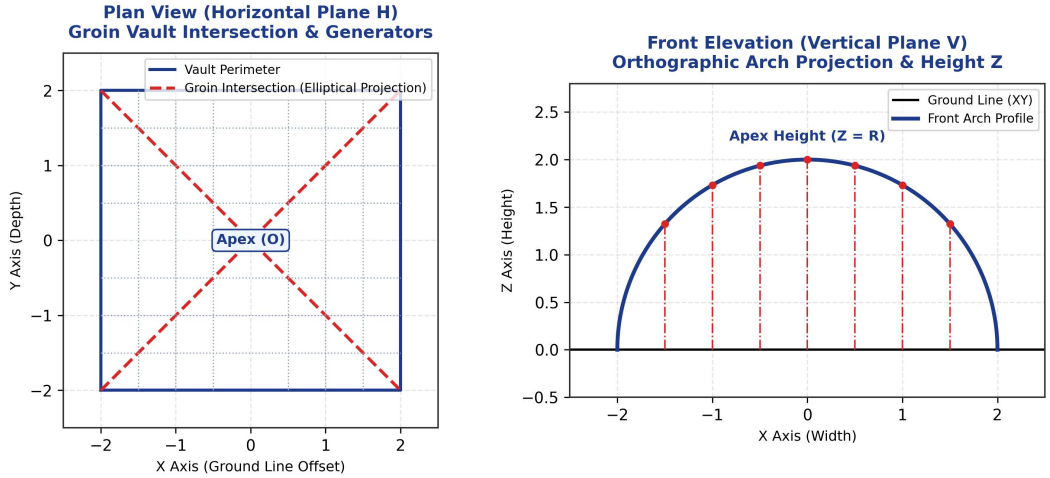


Figure 1: Double orthographic projection of a groin vault intersection showing Plan (Horizontal plane H) and Front Elevation (Vertical plane V) with generator lines and elliptical groin curves.

2.2 Mathematical Formulations of Vault Projections

To ensure rigorous geometric comprehension, the curriculum integrated explicit parametric equations and orthographic projection rules for four principal vault typologies:

- • Barrel Vault: Semi-cylindrical extrusion $x(\theta) = R\cos(\theta)$, $y(v) = v$, $z(\theta) = R\sin(\theta)$. Projectors yield parallel generator lines in Plan H.
- • Groin Vault: Perpendicular intersection of two semi-cylindrical barrels. Intersections project as diagonal ellipses with semi-major axis $R\sqrt{2}$ in Plan H.
- • Spherical Pendentive Dome: Spherical triangles formed by truncating a hemisphere of radius $Rs = a\sqrt{2}$ with four wall planes $x = \pm a$, $y = \pm a$ and a top horizontal ring at height $z = a$.

BG Pulse Journal USA
ISSN:1947-2536 http://www.bgpulseusa.com/index.php/bg

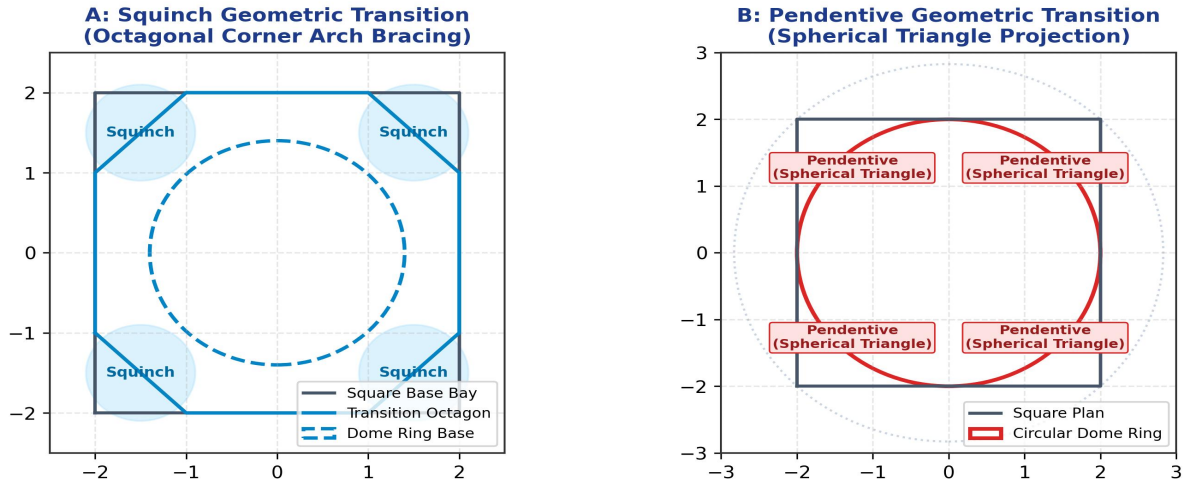


Figure 2: Geometric transition strategies from square bay to circular dome: (A) Octagonal Squinch arch corner bracing; (B) Spherical Pendentive triangle projection.

3. Results

3.1 Spatial Visualization Ability Growth (PSVT:R)

Quantitative pre-test and post-test scores on the PSVT:R assessment revealed remarkable discrepancies across the three instructional cohorts.

Cohort Group	Pre-Test Score	Post-Test Score	Score Gain (Δ)	% Gain
Group A (2D Control)	18.5 ± 3.2	20.8 ± 3.1	+2.3	+12.4%
Group B (Stereotomy)	18.1 ± 3.4	25.0 ± 2.8	+6.9	+38.2%
Group C (Hybrid DG)	18.4 ± 3.1	29.7 ± 1.9	+11.3	+61.6%

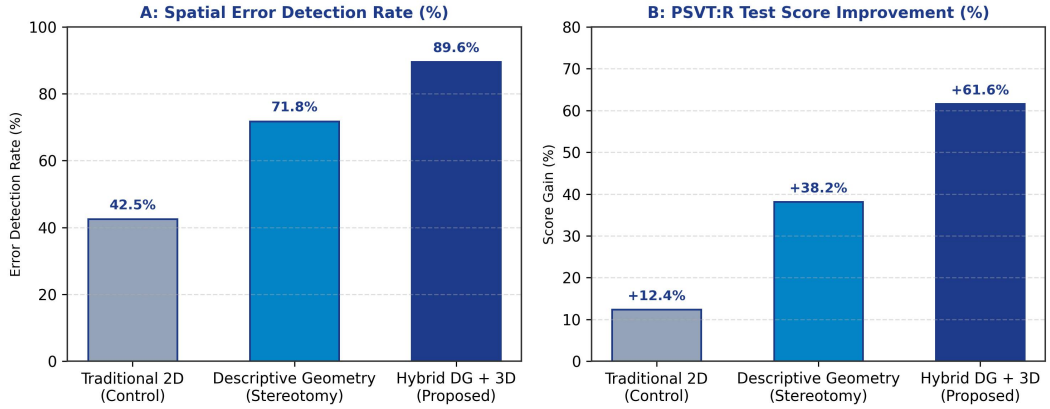


Figure 3: Empirical evaluation metrics across cohorts: (A) Geometric error detection rate (%); (B) PSVT:R spatial visualization score gain (%)

3.2 Comparative Synthesis of Vault Systems

Vault System	Base Plan	Surface Curvature	Orthographic Feature	Group C Mastery
Barrel Vault	Rectangular bay	Single-curved (Cylindrical)	Parallel generators in Plan H; Arch in Elevation V	95.0%
Groin Vault	Square or rectangular	Intersecting Cylinders	Diagonal elliptical groin curves in Plan H	91.2%
Cloister Vault	Square bay	Inward Triangular Webs	Diagonal miter lines meeting at center apex	88.4%
Pendentive Dome	Square to Circular	Double-curved (Spherical)	Spherical triangles bounded by circular arcs	83.8%

4. Discussion

The empirical results provide compelling evidence that Gaspard Monge's double orthographic projection system is not an obsolete historical drafting convention, but a powerful cognitive tool for spatial reasoning. According to Cognitive Load Theory (Sweller, 2011), working memory becomes overwhelmed when novices attempt to process 3D spatial rotations without structured visual scaffolding.

In traditional 2D drafting (Group A), students rely on passive visual memorization of vault elevation templates. When faced with complex 3D intersections (such as pendentive spherical cuts), their lack of structural coordinate systems leads to high error rates (67.6% error oversight). In contrast, Monge's method provides a systematic point-by-point coordinate mapping mechanism. By projecting spatial points onto horizontal (H) and vertical (V) reference planes and using auxiliary elevation views to determine true surface lengths, students build robust mental models of 3D curvature.

5. Conclusion

This study demonstrates that teaching the geometric structure of domes and vaults through descriptive geometry methods significantly enhances spatial visualization abilities and technical graphic precision in architectural education. By combining Gaspard Monge's classical double orthographic projection with modern 3D digital verification, educators can effectively bridge the gap between 2D orthographic documentation and complex 3D vaulted architecture.

References

1. Calvo-López, J. (2020). Stereotomy: Stone Construction and Geometry in Western Europe 1200–1900. Springer Nature Switzerland.

2. Casonato, C. (2016). Art du Trait: Considerations on Double Orthogonal Projection in Medieval Stereotomy. Proceedings of International Conference on Descriptive Geometry, 186–192.
3. Lawrence, S., et al. (2019). Descriptive Geometry, The Spread of a Polytechnic Art: The Legacy of Gaspard Monge. Springer International Publishing.
4. Pütz, C. (2021). Teaching Descriptive Geometry for Architects: Didactic Principles and Effective Methods Demonstrated. RWTH Aachen Institute for Geometry.
5. Gómez-Tone, H. C., et al. (2021). Immersive Virtual Reality for Developing Spatial Ability in Architecture Students. International Journal of Engineering Education, 37(1), 145–157.
6. Heriot-Watt University Research Group. (2024). Enhancing Construction and Architectural Education through 4D BIM and Virtual Reality Integration. Buildings Journal, 14(3116), 1–18.
7. Frontiers in Education. (2024). Enhancing Mathematical and Spatial Education with Spatial Visualization Tools: Revised PSVT:R Assessment Integration. Frontiers in Education, 9(1229126).
8. Studio Matrx Architectural Graphics. (2023). Orthographic Projection & Descriptive Geometry Principles: Fundamentals for Architectural Design.
9. Makhmudov M.M, & Khaitov M.B. (2025). Checking For Condensation Formation on The Inner Surface of The External Walls of Frame Buildings Made of Thermal Blocks. Sustainable Civil Building Management and Engineering Journal, 2(3), 10. https://doi.org/10.47134/scbmej.v2i3.4193
10. Махмудов, М. М., & Хаитов, М. Б. (2025). Проверка на образование конденсата на внутренней поверхности наружных стен каркасных зданий, выполненных из термоблоков. Eurasian Journal of Applied Engineering and Technology (EJAET), 151(2), 9-19.
11. Xaitov Ma'ruf Bolikulovich. (2026). O'ZBEKISTON RESPUBLIKASIDA ENERGIYA SAMARADOR QURILISH UCHUN BAZALT ASOSIDAGI ISSIQLIK IZOLYATSIYA MATERIALLARI [Dataset]. In AKADEMIK TADQIQOTLAR JURNALI (Vol. 3, Issue 01, pp. 186–190). Zenodo. https://doi.org/10.5281/zenodo.22994805
12. Bolikulovich, K. M., & Pulatovich, M. B. (2022). HEAT-SHIELDING QUALITIES AND METHODS FOR ASSESSING THE HEAT-SHIELDING QUALITIES OF WINDOW BLOCKS AND THEIR JUNCTION NODE WITH WALLS. Web of Scientist: International Scientific Research Journal, 3(11), 829-840.
13. Salomovich, T. E., & Pulatovich, M. B. (2026). SOLVING A FINITE DIFFERENCE ONE-DIMENSIONAL PROBLEM USING THE PROGONKA METHOD BASED ON AN IMPLICIT SCHEME. IMRAS, 9(2), 24-30.
14. Rakhimov, A., Rakhimov, L., Khaydarov, S., Beknazarov, M., Achildiyev, R., & Madiev, F. (2024, November). Experimental studies of the pile-in-pipe seismic isolation system with disconnecting connections. In AIP Conference Proceedings (Vol. 3244, No. 1, p. 040020). AIP Publishing LLC.
15. Abduazizovich, R. L. (2021). Din-panah the city of Humayun.