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REGULAR SOLVABILITY OF THE PROBLEM FOR A MIXED PARABOLIC-HYPERBOLIK THIRD-ORDER EQUATION.

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Abstract: The theory of equations of mixed type is one of the central sections of the modern theory of partial differential equations. Interest in these issues is related both to the theoretical significance of the results obtained and to the identification of numerous applied problems, the mathematical modeling of which necessitates the study of various types of equations in the considered domain of independent variable changes. Most of the works in this field have been devoted to the theoretical and applied aspects of equations of mixed elliptic-hyperbolic type. Relatively few studies have been dedicated to the investigation of boundary value problems for equations of mixed parabolic-hyperbolic type. Meanwhile, these equations, like those of the elliptic-hyperbolic type, form the basis of mathematical models of various natural phenomena.We note that the solvability issues and spectral properties of local and nonlocal problems for a mixed parabolic-hyperbolic equation of the second and third orders were studied in
Keywords: problem Koshi, parabolo-hyperbolic equations, The Volterra integral equation of the second kind.

The theory of mixed-type equations is one of the central branches of the modern theory of partial differential equations. The interest in these problems is connected both with the theoretical significance of the results obtained and with the identification of numerous applied problems, whose mathematical modeling necessitates the study of various types of equations in the considered domain of independent variable changes.

The issues of solvability and spectral properties of local and nonlocal problems for mixed parabolic-hyperbolic equations of the second and third orders have been studied in works [1]-[7].

Let $\Omega \quad R^2$ be a finite domain bounded, for $y>0$, by the line segments AA_0, A_0B_0, B_0B of the straight lines $x=0,\ y=1,\ x=1$ respectively;and for $y<0$, by the characteristics $AC:\ x+y=0$ and $BC:\ x-y=1$, of a mixed parabolic–hyperbolic equation of the third order

$$Lu=f((x,y) \qquad (1)$$

where

- $Lu=\dfrac{\partial}{\partial x}\begin{matrix} u_x-u_{yy}, & y>0 \\ u_{xx}-u_{yy}, & y<0 \end{matrix}$

Let $AD:\ y=-\gamma(x),\ 0 \quad x \quad l$, be a smooth curve, where $0{,}5<l<1;\ \gamma(0)=0,\ l+\gamma(l)=1$. The curve is located inside the characteristic triangle $0 \quad x+y \quad x-y \quad 1$. With respect to the curve AD, we further assume that $\gamma(x)$ is a

twice continuously differentiable function, and the functions $x-\gamma(x)$ and $x+\gamma(x)$ are monotonically increasing; that is,

$$0<\gamma(0)<1,\ \gamma(x)>0,\ x>0$$

Problem BM. Find the solution of equation (1) satisfying the boundary conditions

$$u\big|_{AA_0\ A_0B_0}=0, \qquad (2)$$

$$\left.\frac{\partial u}{\partial n}\right|_{AA_0\ AC}=0, \qquad (3)$$

$$\left[u_x-u_y\right][\theta_0(t)]+\mu(t)\left[u_x-u_y\right][\theta^*(t)]=0 \qquad (4)$$

where n is the inward normal, $\theta_0(t)$ and $(\theta^*(t))$ are the affixes (coordinates) of the intersection points of the characteristic AC (of the curve AD) with the characteristic emerging from the point $(t,0,)$ $0<t<1$, and $\mu(t)$ is a given function.

Let $W_2^l(\Omega)$ denote the Sobolev space with the scalar product $(,)_l$, and norm $\|\ \ \|_l$ and $W_2^0(\Omega)\quad L_2(\Omega)$.

A function $u\quad C^1(\bar{\Omega})$ will be called a *regular solution* of problem BM if it possesses continuous derivatives that enter equation (1) in the domains Ω_0 and Ω_1, and in these domains it satisfies equation (1) together with boundary conditions (2) – (4). Here $\Omega_0=\Omega\quad\{y>0\},\ \Omega_1=\Omega\quad\{y<0\}$.

The following theorem is proved

Theorem. Let $\mu(t)\quad C^2[0,1]$ and $\mu(t)\quad -1,\ 0\quad t\quad 1$. Then for any function $f(x,y)\quad C^1(\bar{\Omega}),\ f(A)=0,$ there exists a **unique regular solution** of problem *A* and it satisfies the inequality

$$\|u\|_1\quad c\|f\|_0 \qquad (5)$$

and can be represented in the form

$$u(x,y)=\quad K(x,y,x_1,y_1)f(x_1,y_1)dx_1dy_1\,, \qquad (6)$$

Ω

where $K(x,y,x_1,y_1)\quad L_2(\Omega\quad\Omega)$, and *c* denotes positive constants, possibly different in various cases.

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