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NONLOCAL PROBLEMS FOR A MIXED FRACTIONAL ORDER EQUATION

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Annotation. In recent years, interest has increased in the study of fractional order differential equations in which the unknown function is contained under the sign of the fractional order derivative. This is due both to the development of the theory of fractional integration and differentiation itself, and to applications in various fields of science: physics, mechanics, chemistry, engineering, anomalous diffusion processes and other areas of natural science.

Since an equation of fractional order generalizes an equation of integer order, as well as a relatively small number of systematic analytical and numerical methods for such equations, this direction is made a priority for the general theory of differential equations.

The spectral properties, including the Volterra property of a mixed fractional order equation, have, as far as we know, been almost unstudied.

This work is devoted to one of the most important problems - the study of the solvability and spectral properties (Volterra property) of three nonlocal problems for the diffusion-hyperbolic equation (fractional order).

Consider the equation

$$Lw(x,y)=f(x,y), \qquad (1)$$

Where

$$Lw(x,y)=\begin{cases} D_{0x}^{\alpha}w(x,y)-w_{yy}(x,y), & (x,y)\ \ \Omega_0 \\ w_{xx}(x,y)-w_{yy}(x,y), & (x,y)\ \ \Omega_1 \end{cases} \qquad (2)$$

$$D_{0x}^{\alpha}w(x,y)=\frac{1}{\Gamma(1-\alpha)}\int_{1}^{x}\frac{w_x(t,y)}{(x-t)^{\alpha}}dt, \quad 1<\alpha<2$$

$\Gamma(x)$ – Euler's gamma function, (2) – integro-differential operator of fractional order α in Caputo's sense [1, c.92], in the area $\Omega=\Omega_0 \ \ \Omega_1 \ \ AB$.

Здесь Ω_0 - rectangle ABB_0A_0 с peaks $A(0,0),B(1,0),B_0(1,1),A_0(0,1),\Omega_1$ – area bounded by segments AB and characteristics $AC:x+y=0,\ \ BC:x-y=1$ equations (1.1), $f(x,y)$ - given function. Let the smooth curve $AD:y=-\gamma(x),\ 0<x<l,$ где $0,5<l \ \ 1;\ \gamma(0)=0,\ l+\gamma(l)=1,$ If $l<1$ и $\gamma(l)=0$, If $l=1$ located inside the characteristic triangle $0<x+y \ \ x-y<1.$

Relative to the curve $\gamma(x)$, suppose that $\gamma(x)$ – twice continuously differentiable function and $x\pm\gamma(x)$ – monotonically increasing functions, and $0<\gamma'(x)<1,\ \gamma(x)>0,\ x>0$.

Task B. Find a solution to equation (1.1) that satisfies the conditions:

$$w(0,y)=0, \quad 0 \quad y \quad 1, \qquad (3)$$

$$w_x(0,y)=0, \quad 0 \quad y \quad 1, \qquad (4)$$

$$w(x,1)=0, \quad 0 \quad x \quad 1, \qquad (5)$$

$$[w_x - w_y][\theta_0(t)] + \mu(t)[w_x - w_y][\theta^*(t)] = 0, \quad 0 < t < 1, \qquad (6)$$

Where $\theta_0(t), (\theta^*(t)) -$ affix of the point of intersection of the characteristic AC (crooked AD) c characteristic emanating from the point $(t,0)$, 0<t<1, $\mu(t)$ – given function.

In the area Ω_0 consider the following auxiliary problem

Task B1. Find a solution to equation (1) at $y > 0$, satisfying the conditions (3), (4), (5) And

$$w_x(x,0) - w_y(x,0) = h(x), \quad 0 \quad x \quad 1, \qquad (7)$$

Where $h(x)$ – given function.

Lemma 1. Let $h(x)$ $C^1[0,1]$. Then for any function $f(x,y)$ $C^1(\bar{\Omega}_0)$ the solution of the problem allows for an a priori estimate.

$$D_{ox}^{\alpha-1}\|w(x,y)\|^2_{L_2(0,1)} + 2\int_1^x \|w_y(t,y)\|^2_{L_2(0,1)}dt \quad k\int_1^x \|f(t,y)\|^2_{L_2(0,1)}dx + \int_1^x h^2(t)dt, \qquad (8)$$

Where $\|f(x,y)\|^2_{L_2(0,1)} = \int_1^2 f^2(x,y)dy$

Here and below the symbol will denote a positive constant independent of $w(x,y)$, not necessarily the same one.

Lemma 2. Let $\delta(x)$ $C^1[0,1]$ And $\delta(x)$ -1. Then for any function $f(x,y)$ $C^1(\bar{\Omega})$, $f(0,0)=0$ problem solution E1B admits an a priori estimate

$$D_{ox}^{\alpha-1}\|w(x,y)\|^2_{L_2(0,1)} + \int_1^x \|w_y(t,y)\|^2_{L_2(0,1)}dt \quad k\int_1^x \|f(t,y)\|^2_{L_2(0,1)}dt + \int_1^x d\xi \int_\xi^x |f(\xi,t)|^2 dt \qquad (9)$$

Lemma 2 implies the following estimate:

$$\|w(x,y)\|_{L_2(\Omega_0)} + \|w_y(x,y)\|_{L_2(\Omega_0)} \quad k\|f(x,y)\|_{L_2(\Omega)}, \qquad (10)$$

Where $L_2(\Omega)$ - space of square summables in Ω functions.

Thus, we have proven the following theorem

Теорема 2.1. Пусть $\delta(x)$ $C^1[0,1]$ и $\delta(x)$ $-1, 0$ x 1. . Then for any function $f(x,y)$ $C^1(\bar{\Omega}), f(A)=0$ there is a unique regular solution to the problem E1B (1.1), (2.1)-(2.4) and it can be represented in the form (2.38) and satisfies the inequality (2.43).

LITERATURE

1. Kilbas A.A., Srivastava H.M., Trujillo J.J. Theory and applications of fractional differential equations (North-Holland Mathematics Studies, 204). - Amsterdam: Elsevier, 2006. - 523 p.

2. Псху А.В. Уравнения в частных производных дробного порядка. - М.: Наука, 2005. - 199 с.

3. Hilfer R., Luchko Y., Tomovski Z. Operational method for solution of the fractional differential equations with the generalized Riemann-Liouville fractional derivatives // Fract. Calculus Appl. Anal.- DeGruyter, 2009. –v.12. -P.299-318.

4. Berdyshev, A.S., Cabada, A., Turmetov, B.K. On solvability of a boundary value problem for a nonhomogeneous biharmonic equation with a boundary operator of a fractional order // Acta Mathematica Scientia, 2014, 34(6), стр. 1695–1706. doi: 10.1016/S0252-9602(14)60115-6.

5. Berdyshev A.S. The volterra property of some problems with the Bitsadze-Samarskii-type conditions for a mixed parabolic-hyperbolic equation // Siberian Mathematical Journal, 2005, 46(3), стр. 386–395. doi: 10.1007/s11202-005-0041-y.

6. Berdyshev, A.S., Cabada, A., Karimov, E.T., Akhtaeva, N.S. On the Volterra property of a boundary problem with integral gluing condition for a mixed parabolic-hyperbolic equation // Boundary Value Problems, 2013, 2013, 94. doi: 10.1186/1687-2770-2013-94.

7. A.S. Berdyshev, Boundary value problems and their spectral properties for the equation of mixed parabolic-hyperbolic and mixed-composite types / Abai Kazakh National Pedagogical University, Institute of Information and Computing Technologies. - Almaty, 2015.-224p. (in Russian)

8. Karimov E.T. Boundary value problems with integral transmitting conditions and inverse problems for integer and Fractional order differential equations. Information-resource centre of V.I.Romanovkiy Institute of Mathematics, Tashkent, 2020.