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ABOUT ONE PROBLEM WITH AN OBLIQUE DERIVATIVE FOR A MIXED EQUATION.

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Annotation. The main goal of this work is to show that the correctness of the problem with an oblique derivative specified at a departure from the characteristic for a mixed parabolic-hyperbolic equation with a characteristic line of changing type significantly depends on the class in which the solution is sought and on the value of the polar angle formed by the vector field defining the oblique derivative and the abscissa axis at the beginning of the departure.

Let $\Omega \quad R^2$ - finite area limited by y>0 in segments AA_0, A_0B_0, BB_0, A(0,0), $A_0(0,1)$, B(1,0), $B_0(1,1)$, a at y<0 - smooth curve $AD: y = -\gamma(x), 0 \quad x \quad \ell$, Where $0{,}5 < \ell \quad 1, \ \gamma(0) = 0, \ \ell + \gamma(\ell) = 1,$ located inside the characteristic triangle $ABC: 0 \quad x + y < x - y \quad 1$, and a segment of characteristics $BC: x - y = 1, \ \ell \quad x \quad 1$, equations of mixed parabolic-hyperbolic type

$$Lu = f(x, y), \qquad (1)$$

Where

$$Lu = \begin{cases} u_{xx} - u_y, & y > 0 \\ u_{xx} - u_{yy}, & y < 0. \end{cases}$$

Regarding the AD curve, we will assume that at no point does it have a characteristic direction and belongs to the class of functions $C^1[0,\ell]$ и $\gamma(0) = 0, \quad 0 < x \quad \ell < 1, \quad \ell + \gamma(\ell) = 1,$ function $x + \gamma(x)$ increases monotonically.

BM task. Find a solution to equation (1) that satisfies the conditions

$$u\big|_{AA_0 \quad BB_0} = 0, \qquad (2)$$

$$\left[a(x,y)u_x + b(x,y)u_y + c(x,y)u\right]_{AD} = 0, \qquad (3)$$

Where $a(x,y), b(x,y), c(x,y)$ - specified functions. For regions D, such that $\overline{D} \quad \{A\} \quad \varnothing$, let's define a function space

$$C_\alpha^{m.n}\left(\overline{D}\right) = \left\{ u(x,y) : \frac{\partial^{i+j}u(x,y)}{\partial x^i \partial y^j} \quad C(\overline{D}), \ \frac{\partial^{i+j}u(0,0)}{\partial x^i \partial y^j} = 0, \ i + \frac{m}{n} j \quad m-1, \ m \quad n \quad 0, \right.$$

$$\left. \frac{\partial^{i+j}u(x,y)}{\partial x^i \partial y^j} \quad C(\overline{D} \setminus A), \ i + \frac{m}{n} j = m, \ \max_{i+\frac{m}{n}j=m} Sup_{x \ \overline{D}} \left| x^{-\alpha} \frac{\partial^{i+j}u(x,y)}{\partial x^i \partial y^j} \right| < \ , \alpha \quad 0 \right\}.$$

$$\|u\|_{m,n,\alpha} = \max_{i+\frac{m}{n}j=m} Sup_{(x,y) \ \overline{D}} \left| x^{-\alpha} \frac{\partial^{i+j}u(x,y)}{\partial x^i \partial y^j} \right|$$

It is clear that relative to the norm space $C_\alpha^{m.n}\left(\overline{D}\right)$ is Banach.
We similarly define the space on the segment [0,1]:

$C_\alpha^k[0,1] = \varphi(x) : \varphi^{(j)}(x) \quad C[0,1],\ \varphi^{(j)}(0) = 0, j = 0,1..k-1.\varphi^{(k)}(x) \quad C(0,1], \underset{0\ x\ 1}{Sup} x^{-\alpha}\left|\varphi^{(k)}(x)\right| <$

with the norm

$$\left\|\varphi(x)\right\|_{k,\alpha} = \underset{0\ x\ 1}{Sup}\left|\frac{\varphi^{(k))}(x)}{x^\alpha}\right|,\ k \quad 0, \alpha \quad 0.$$

Let $\Omega_0 = \Omega \quad \{y > 0\},\ \Omega_1 = \Omega \bigcap \{y < o\}.$

By a regular solution to the TM problem we mean a bounded function from the class $C(\overline{\Omega} \setminus B) \quad C^1(\Omega \quad AD) \quad C^{2.1}(\Omega_0) \quad C^{2.2}(\Omega_1),$

satisfying the equation (1) V $\Omega_0 \quad \Omega_1$ and conditions (2) и (3).

Let us denote, as usual, $\tau(x) = u(x,0),\ \nu(x) = u_y(x,0).$ (4)

Due to the unique solvability of the first initial-boundary value problem for the heat equation, any regular solution to equation (1) with y>0, satisfying the conditions (2) и $u\big|_{AB} = \tau(x)$, representable in the form

$$u(x,y) = -\int_{0\ 0}^{y\ 1} G(x,x_1,y-y_1)\, f(x_1,y_1)dx_1dy_1 + \int_0^1 G(x,x_1,y)\tau(x_1)dx_1, \quad (5)$$

Where $G(x,x_1,y)$ - Green's function of the first boundary value problem for equation (1) in region Ω_0:

$$G(x,x_1,y) = 2 \quad e^{-\pi^2 n^2 y}\ Sinn\pi\ Sinn\pi x_1$$

In the area Ω_0 in equation (1), passing to the limit at $y \to 0$, taking into account (4), we get

$$\tau''(x) - \nu(x) = f(x,0), \quad 0 < x < 1. \quad (6)$$

Relationship (6) is the main functional relationship between $\tau(x)$ And $\nu(x)$, brought to segment AB from the region Ω_0.

Further, due to the unique solvability of the Cauchy problem for the wave equation, any regular solution of equation (1) with y<0 representable in the form

$$u(\xi,\eta) = \frac{1}{2}\ \tau(\xi) + \tau(\eta) - \int_\xi^\eta \nu(t)dt \ - \int_\xi^\eta d\xi_1 \int_{\xi_1}^\eta f_1(\xi_1,\eta_1)d\xi_1 d\eta_1, \quad (7)$$

где

$$\xi = x + y,\ \eta = x - y,\ f_1(\xi,\eta) = \frac{1}{4} f\ \frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}.$$

In formula (7), satisfying condition (3), we have

$$[a(x) - b(x)][\tau'(x) - \nu(x)] + [a(x) + b(x)][\tau'(\lambda(x) + \nu(\lambda(x))] + c(x)\ \tau(x) + \tau(\lambda(x)) - \int_{\lambda(x)}^x \nu(t)dt\ = F(x),$$

0<x<1, (8)

$$F(x)=2c(x)\int_{\lambda(x)}^{x}d\xi_1\int_{\xi_1}^{x}f_1(\xi_1,\eta_1)d\eta_1+[a(x)-b(x)]\int_{\lambda(x)}^{x}f_1(\xi_1,x)d\xi_1-\ a(x)+b(x)\int_{\lambda(x)}^{x}f_1(\lambda(x),\eta_1)d\eta_1. \quad (9)$$

Здесь $\xi=\lambda(\eta)$ - curve equation AD in characteristic variables ξ,η. Due to the conditions imposed on AD, we have $\lambda(x)<x,\ 0<x<1,\lambda(0)=0,\ \lambda'(x)>0,\ \lambda'(0)<1.$

Relationship (8) is the basic relationship between $\tau(x)$ And $\nu(x),$ brought to segment AB from the region Ω_1.

Let $a(x,y)$ $b(x,y)$. Through θ Let us denote the polar angle formed by the vector field defining the oblique derivative and the x-axis at the beginning of the departure (at point A).

Theorem. If $-\frac{\pi}{2}<\theta<0$ or $\frac{\pi}{2}<\theta<\pi$, then for any function $f(x,y)$ $C_{\alpha}^{0.0}(\overline{\Omega}_0)$ $C_{\alpha}^{1.1}(\overline{\Omega}_1)$ $C^{1.0}(\Omega_0)$ there is a unique regular solution to the BM problem from the class $u(x,y)$ $C^{1.1}(\overline{\Omega}\setminus B)$ $C_{\alpha}^{2.1}(\overline{\Omega}_0\setminus B)$ $C_{\alpha}^{2.2}(\overline{\Omega}_1)$.

If $0<\theta<\frac{\pi}{2}$ or $\pi<\theta<\frac{3\pi}{2}$, then the BM problem in the indicated class is Hausdorff normally solvable and its index χ=+ and, in particular, the corresponding homogeneous problem has an infinite number of linearly independent solutions.

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