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ABOUT ONE PROBLEM FOR A THIRD ORDER EQUATION

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Annotation. The work is devoted to the study of questions of regular and strong solvability of problems with conditions of the Bitsadze–Samarsky type for a third-order mixed parabola–hyperbolic equation with two independent variables.

Keywords. Cauchy problem, parabolic-hyperbolic equation, Volterra integral equation of the second kind, S.L. Sobolev space with scalar product, strong solutions.

Questions of solvability and spectral properties of local and nonlocal problems for mixed parabolic-hyperbolic equations of the second and third orders were studied in [1]-[3], for various equations of mixed type of fractional order were considered in [4]-[6].

Consider the mixed parabolic-hyperbolic equation of third order

$$Lu = f((x,y) \qquad (1)$$

Where

$$Lu = \frac{\partial}{\partial x} \begin{matrix} u_x - u_{yy}, & y > 0 \\ u_{xx} - u_{yy}, & y < 0 \end{matrix}$$

Let $\Omega \quad R^2$ - finite area limited by $y > 0$ in segments AA_0, A_0B_0, B_0B direct $x = 0,\ y = 1,\ x = 1$ accordingly, and when $y < 0$ characteristics $AC:\ x + y = 0$ And $BC:\ x - y = 1$, smooth curve $AD:\ y = -\gamma(x),\ 0 \quad x \quad l$, Where $0{,}5 < l < 1;\ \gamma(0) = 0,\ l + \gamma(l) = 1$ located inside the characteristic triangle $0 \quad x + y \quad x - y \quad 1$. Relative to curve AD Throughout what follows we assume that $\gamma(x)$ - twice continuously differentiable function and functions $x - \gamma(x)$ And $x + \gamma(x)$ increase monotonically;

$$0 < \gamma'(0) < 1,\ \gamma(x) > 0,\ x > 0$$

Task B. Find a solution to equation (1) that satisfies the boundary conditions $u\big|_{AA_0 \quad A_0B_0} = 0,$

(2)

$$\left.\frac{\partial u}{\partial n}\right|_{AA_0 \quad AC} = 0, \qquad (3)$$

$$[u_x - u_y][\theta_0(t)] + \mu(t)[u_x - u_y][\theta^*(t)] = 0 \qquad (4)$$

Where n - internal normal, $\theta_0(t)$ $(\theta^*(t))$ - affix of the point of intersection of the characteristic AC (crooked AD) with a characteristic coming from the point $(t,0,)$ $0 < t < 1,\ \mu(t)$ - given function.

Through $W_2^l(\Omega)$ let us denote the space of S.L. Sobolev with the scalar product $(,)_l$, and the norm $\| \quad \|_l$, $W_2^0(\Omega) \quad L_2(\Omega)$.

Theorem 1. Let $\mu(t) \quad C^2[0,1]$ And $\mu(t) \quad -1, \ 0 \quad t \quad 1$. Then for any function $f(x,y) \quad C^1(\overline{\Omega}), \ f(A)=0$ there is a unique regular solution to Problem B and it satisfies the inequality

$$\|u\|_1 \quad c\|f\|_0 \qquad (5)$$

and can be represented in the form

$$u(x,y) = \int_{\Omega} K(x,y,x_1,y_1) f(x_1,y_1) dx_1 dy_1, \qquad (6)$$

Where $K(x,y,x_1,y_1) \quad L_2(\Omega \quad \Omega)$, c – hereinafter means positive, generally speaking, different constants. Through W let us denote the set of functions from the class

$$u \quad C^1(\overline{\Omega}), \ u_{xx}, u_{yy} \quad C(\overline{\Omega}_0), \ u_{xxx}, u_{yyy} \quad C(\overline{\Omega}_1)$$

satisfying boundary conditions (2) – (4).

Function $u \quad L_2(\Omega)$ we call a strong solution to Problem B if there is a sequence of functions $\{u_n\}, u_n \quad W$, such that u_n и Lu_n converge in $L_2(\Omega)$ to and f respectively.

Theorem 2. For anyone $f(x,y) \quad L_2(\Omega)$ there is only one strong solution problem B. This solution belongs to the class $C(\overline{\Omega}) \quad W_2^1(\Omega)$, satisfies inequality (5) and can be represented in the form (6).

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