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CONFIGURATIONS ON THE GRID

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Configurations on a Lattice

Abstract: This paper studies configurations on a lattice and their mathematical properties. The configuration space and Gibbs measure based models in lattice systems are analyzed.

Keywords: lattice, configuration, lattice system, Gibbs measure, statistical mechanics.

Configurations on the grid. A configuration on a lattice is one in which a value (such as a spin state) is assigned to each node on the lattice. If each node accepts only two values (for example, +1 or -1), this is called an Izing model.

If there are more values (eg 3 or more), then it is a Potts model.

Formally, the configuration space is expressed as:

$\Omega = \{\sigma : V \to \Phi\}$,

here:

- V — the set of nodes of the lattice,
- Φ — possible cases (eg, {−1, +1}).

Lattice interactions and energy functions.

An energy function is introduced to represent the interactions between particles. This energy is determined by the Hamiltonian function.

For example, in the Izing model, the interaction between two neighboring nodes is as follows:

$$H(y) = -J \sum_{<x,y>} y(x)y(y),$$

here: $\sum_{<x,y>}$ denotes the sum over all adjacent pairs,

- J — influence,
- σ(x) and σ(y) — x and y spins at nodes.

If J > 0 if so, this is a ferromagnetic model, J < 0 If there is, this is an antiferromagnetic model. We will describe the Izing model in the next sections.

Hamiltonian concept.

The Hamiltonian is a function that represents the total energy of the system. It depends on the configuration s and is often written as a sum of local interactions.

For the tracing model, this is written as:

$$H(y) = -J \sum_{<x,y>} y(x)y(y) - h \sum_{x \in V} y(x),$$

here:

h — external magnetic field.

The Hamiltonian determines the main properties of a physical system, since equilibrium states are considered exactly as minimum energy states.

Graphic images (Z^2 fence and Keli trees)

ZI before we tell you about the fence Z^1 - let's get acquainted with the lattice (we will not make a mistake even if we take it as a 1st-order keli tree):

Description: Z^1 - is a one-dimensional integer lattice, that is, the whole set of all numbers, each point of which has only two neighbors: x-1 on the left and x+1 on the right.

Formal writing:

$Z^1 = x \, OZ$

Z- the set of integers.

Neighbor Set:

$N(x) = \{x-1, x+1\}$

Example:

If x=3, then:

$N(3) = \{2,3\}$

...,-2,-1,0,1,2,3,4,5,...

Graphic View (illustrative):

$\ldots — (-2) — (-1) — (0) — (1) — (2) — (3) — (4) — \ldots$

Here, each point is an integer, and the lines represent their neighbor relationship.

Z^1 - lattice Izing model, Potts model and other lattice physics models are simple but important basis.

For example, in the Izing model, each point has a spin state $(y(x) = \pm 1)$ and only interact with neighbors.

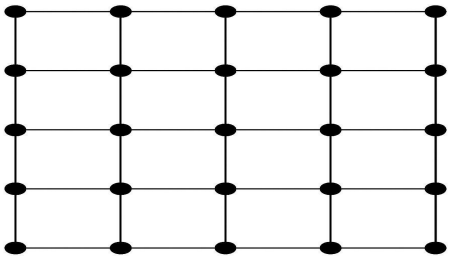
ZI A lattice is a two-dimensional regular array where each point is represented as (x,y) where x and y are integers.

Each point has 4 neighboring points: left, right, top, bottom.

This lattice is one of the most commonly used examples in the trace model and other lattice systems. This lattice is a fundamental structure in the study of two-dimensional physical models.

Neighborhood relationship. Each point (i,j) is adjacent to the following points: $(i+1, j), (i-1, j), (i, j+1), (i, j-1)$

Here, the nodes i and j are denoted and they are taken from the set of integers:

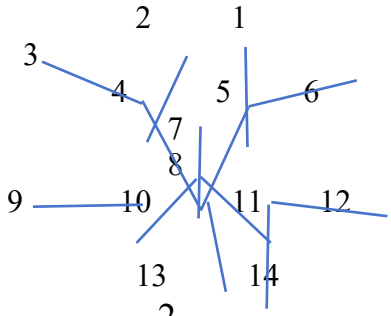


The intersection of each straight line (ie, each black dot is a grid node, and the lines connecting them are neighbors.) ZI is called a node.

Physical meaning:

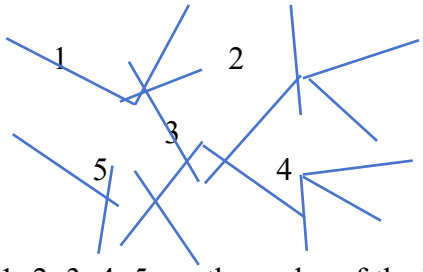
In the Izing model, this means that spin particles are located at each node of the lattice. Each particle interacts only with its immediate neighbors.

A Keli tree is a graph without cycles, and each node has the same degree (for example, if each node has 3 neighbors, the tree is of order 2). Exact solutions of models can be found using recursive formulas on such trees:



The given graph is a tree of order 2, i.e Γ^2.

Here, 1,2,...,14 are called the vertices of the tree.



This is a 3rd order keli tree and 1, 2, 3, 4, 5 are the nodes of the keli tree. These two graphical structures are widely used in studying various properties of lattice systems.

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