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BASE CASES FOR THE MODIFIED EXTERNAL FIELD POTTS MODEL

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Ground states for the Potts model with a modified external field

Abstract: the ground states of the Potts model with a modified external field on the Z^2 lattice have been studied.

Keywords: Z^2 lattice system, configuration, Potts model with a modified external field, ground states.

Let's assume $Z^2 \to R^2$ be a lattice of ordinary integers in and $\Phi = \{0,1,2....,q\}$ let it be Configuration is a set of values Φ taken from $g(i)$, i Z^2 function is called, i.e $g(i)$ X. A collection of all configurations m we define with . m phase shift groups move at $g(i)$ configuration with a finite index Z Z^2 is translational-invariant with respect to the displacement group, then this configuration is called a periodic configuration. If the configuration is invariant with respect to integer vectors with even coordinates, then we say that its period does not exceed. It overlaps almost everywhere, that is, everywhere except for a finite number s and s' for configurations $H(s,s',J)$ we consider the relative Hamiltonian. $H(s,s',J)$ Hamiltonian s and s' denotes the energy of configurations and is defined as: $H(s,s,J) = \sum_{<i,j>} J(\delta_{ij}) + \sum_{i\ \Phi} \alpha(i \bmod 2)$,

(1)

here $J = (J,\alpha)$ R^2 - optional fixed parameter, $< i, j >$ close neighbors; i, j Z^2.

Definition 1. *A periodic configuration is called the ground state for a relativistic Hamiltonian, if for an arbitrary configuration that overlaps almost everywhere with* $H(a,s)$ 0 *if*

Our goal is voluntary J R^2 is to find the complete set of ground states for

Let's assume M ends through Z^2 we define the set of unit squares with edge length 1 obtained from k M configuration limit in the cube s_k is called a bounded configuration. Two are limited s_k and $s_{k'}^{'}$ Configurations belong to the same class, if one of them is formed by changing the pointers of the other if possible. There are 7 classes for this case. η_l l =1,2,3,…,7 through l - define class configurations. **Lemma 1.** Optional η_l class and optional s_k η_l is periodic with a period not exceeding 2 for a bounded configuration s There is a configuration for them $s_{k'}^{'}$ η_l optional k' M va $s_k^{'} = s_k$ s optional k as energy squared

$$U(s_k;J) = \frac{J}{2} \sum_{<i,j>} (\delta_{ij}) + \frac{\alpha}{k+2} \sum_{i\ \Phi} \alpha(i \bmod 2), \qquad (2)$$

value is said. This is the placeda i and $j-k$ square ends, (J,α) R^2.

δ_{ij} The Kronecker symbol is defined as follows:

$$\delta_{ij} = \begin{matrix} 1, \text{ agar } i=j \text{ bo'lsa,} \\ 0, \text{ agar } i \quad j \text{ bo'lsa,} \end{matrix}$$

The utility of the above value (1) expresses the relative efficiency in the following simple form:

$$H(s,s') = \sum_{k \ M} (U(s_k) - U(s_k')), \qquad (3)$$

$U(s_k)$ each function η_l takes a uniform value in the class, that is, if s_k $\eta_l, s_{k'}'$ η_l if so $U(s_k) = U(s'_k)$. that is why U_l through $U(s_k)$ we set the value of the function in the -class.

If the configuration s such that $U(s) = U_l$ something is fixed l and optional k M for , then the additive constant overlaps with the allocated energy in numerical precision.

V a set of unit squares formed from a set of vertices M we define with . b M the configuration in the square x $b \to \sigma(x)$ Φ we define as a function and it $\sigma(V)$ let's define by. b M the center of the square c_b let's define with . σ_b we calculate the configuration energy using the following formula: $U(\sigma_b)$ $U(\sigma_b, J, \alpha) = -\frac{1}{2} J \sum_{\substack{x,y: \\ x,y \ b}} \delta_{\sigma(x)\sigma(y)} - \frac{\alpha}{k+2} \sum_{x \ b} \sigma(x) \bmod 2$ (4)

here (J,α) R^2.

If $k=2$ and $q=2$ oath σ_b the following is relevant for the configuration:

$U(\sigma_b)$ $\{U_i : i = 1...7\}$

Here

$$U_1 = -\frac{J}{2} - \frac{\alpha}{4},\ U_2 = -\frac{3}{2}J,\ U_3 = -J,\ U_4 = -\frac{\alpha}{4},\ U_5 = -\frac{J}{2} - \frac{\alpha}{4},$$

$$U_6 = -\frac{J}{2},\ U_7 = -J - \frac{\alpha}{4}.$$

Lemma 2. s periodic configuration H is a necessary and sufficient condition for a relative Hamiltonian to be the ground state

$$U(s_k) = \min(\{U_l\}, l = 1,2,...,7), \qquad (5)$$

to be. Here $\min(\{U_l\}, l = 1,2,...,7) = \min U_l$, $U_l(J)$ value J Z^2 will be a linear function depending on the parameter. Each fixed $m = 1,2....,7$ for A_m through so J define a set of points for which

$$U_m(J) = \min(\{U_l(J)\}, l = 1,2,...,7), \qquad (6)$$

Using simple calculations, we can get the following results: $A_1 = \{(J,\alpha)$ $R^2 \mid J$ $0; \alpha$ $0\}$, $A_2 = A_3 = \{(J,\alpha)$ $R^2 \mid J = 0; \alpha$ $0\}$,

$A_4 = \{(J,\alpha)$ $R^2 \mid J$ $0 va \alpha$ $0; J$ $\alpha/2 va \alpha$ $0\}$,

$A_5 = A_6 = A_7 = \{(J,\alpha)$ $R^2 \mid J = 0; \alpha$ $0\}$

and $\bigcup_{i=1}^{7} A_i = R^2$.

Each J R^2 for the point $A_1, A_2, ..., A_7$ two sets $A_{m_1}, A_{m_2}, ..., A_{m_r}$ va $A_{m_{r+1}}, A_{m_{r+2}}, ..., A_{m_7}$ we divide into classes in such a way that

$$J \bigcap_{q=1}^{r} A_{m_a} \quad \text{va} \quad J \bigcup_{q=r+1}^{M} A_{m_q}, \qquad (7)$$

here 1 r 7; m_q $\{1,2,,.....,7\}$ m_q m_p if $p+q$, q $\{1,2....,7\}$

[4] non-periodic ground states for the Potts model with an external field are studied in the work. Using the methods in this article, the following theorem is proved.

Theorem 1. Z^2 For the lattice-modified Potts model with an external field, the following assertion holds:

If (J,α) A_1 if so, then $\sigma(x)=1, x$ V va $\sigma(x)=2, x$ V configurations will be a translation-invariant ground state.

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