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APPLICATION OF HIGH-ORDER DIFFERENTIAL EQUATIONS

Department of Mathematics,
University
of Economics and Pedagogy
Assistant: **Jamalov Madamin Habibulla ugli**
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Higher-Order Linear Differential Equations with Constant Coefficients

Abstract: The article briefly discusses higher-order linear differential equations with constant coefficients and their solution methods.
Keywords: Higher-order linear differential equation, constant coefficients, characteristic equation.

INTRODUCTION

Differential equations are one of the important branches of mathematical analysis and are widely used in modeling natural and technical processes. In particular, high-order linear differential equations with invariant coefficients are important in expressing many problems in mechanics, physics, electrical engineering, and other fields.

Finding a general solution for this type of equations is based on constructing a characteristic equation and determining its roots. The appearance of the solution is determined depending on whether the roots are real, complex or multiple.

In this article, the main theoretical aspects of linear differential equations with high-order constant coefficients and their solution methods are considered.[1]

n-order linear differential equation:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y^1 + a_0 y = g(x) \qquad (1)$$

has a view; where y=y(x) is an unknown function, given $c_1, c_2, \ldots, c_n$ the coefficients are arbitrary real numbers, the free term g(x) is defined in some interval I and is assumed to be continuous. (1) To solve differential equations of the form, we derive the corresponding homogeneous equation:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y^1 + a_0 y = 0 \quad (2)$$

(2) Let's solve the homogeneous differential equation.

This

$$L(a) = a_n a^n + a_{n-1} a^{n-1} + \ldots + a_1 a + a_0 = 0 \quad (3)$$

Equation (2) is called the characteristic equation of the differential equation. The root of the characteristic equation is called the characteristic number: $a_1, a_2, a_3, \ldots, a_n$.

(2) The solution of the homogeneous differential equation is as follows (if our terms consist of real numbers):

$$y = c_1 e^{a_1 x} + c_2 e^{a_2 x} + c_3 e^{a_3 x} + \ldots + c_n e^{a_n x}$$

In this $c_1, c_2, ..., c_n$ arbitrary real numbers.

If the characteristic numbers of our characteristic equation are not real numbers, but complex numbers, i.e.: $a_1 = a_1 \pm ib_1, a_2 = a_2 \pm ib_2, a_3 = a_3 \pm ib_3, ..., a_n = a_n \pm ib_n$.

If so, then the general solution of the differential equation (2) is:

$$y = c_1 e^{a_1 x} \cos b_1 x + c_2 e^{a_1 x} \sin b_1 x + c_3 e^{a_2 x} \cos b_2 x + c_4 e^{a_2 x} \sin b_2 x ... + c_{n-1} e^{\frac{a_n x}{2}} \cos \frac{b_n}{2} x + c_n e^{\frac{a_n x}{2}} \sin \frac{b_n}{2} x.$$

It will be.

Now we find the general solution of the differential equation (1). In some special forms of the right-hand side f(x), the particular solution can be found using the method of unknown coefficients. Right side

$$g(x) = e^{ax}(P_1(x)\cos bx + P_2(x)\sin bx)$$

Let it consist of a quasi-quad, i.e $a_n y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y^1 + a_0 y = e^{ax}(P_1(x)\cos bx + P_2(x)\sin bx)$ (4)

Let's look at the differential equation, where a, b are real numbers; $P_1(x), P_2(x)$ polynomials with real coefficients. Custom solution. To construct the particular solution of equation (4), we proceed as follows.

Let's construct the complex number a+ib for the considered equation (4).

1) If the number a+ib is not a root of the characteristic equation (3), then the particular solution of (4) is:

$$y = e^{ax}(Q_1(x)\cos bx + Q_2(x)\sin bx) \quad \text{(a)}$$

can be searched in appearance, in this $Q_1(x), Q_2(x)$ polynomials and $\max(\deg Q_1(x), \deg Q_2(x)) = \max(\deg P_1(x), \deg P_2(x))$, degQ with Q multi-

the level of the limit is set.

2) If the complex number a+ib is the k-fold root of the characteristic equation (3), then the particular solution of (4) is:

$$y = x^k e^{ax}(Q_1(x)\cos bx + Q_2(x)\sin bx) \quad \text{(b)}$$

can be searched by appearance.

So, it seems to us now that it is enough to add the special solution (a) (or (b)) to the solution of the homogeneous differential equation (2) corresponding to the solution of the differential equation of the form (1). [1]

DISCUSSION

Solve this differential equation:

$$y^{II} + 5y^{I} + 6y = \sin x$$

so let's create a homogeneous differential equation corresponding to our equation: $y^{II} + 5y^{I} + 6y = 0$

Now, in order to solve this homogeneous differential equation, let's formulate and solve the corresponding characteristic equation:

$$a^2 + 5a + 6 = 0$$

we know that this is the solution of the characteristic equation:

$$a_1 = -2, a_2 = -3$$

will be. So the solution of the homogeneous differential equation is: $y = c_1 e^{-2x} + c_2 e^{-3x}$.

Now we find the particular solution of the equation. Let's construct a complex number for the sin4x function: a+ib=i. we know that this complex number is not a solution of the given characteristic equation, so it is a particular solution of the differential equation

$$y = e^{ax}(Q_1(x)\cos bx + Q_2(x)\sin bx) = A\cos x + B\sin x \qquad \text{(c)}$$

We look for it.

We will need it: y, y^{I}, y^{II}.

So, we find the requested information and pour it into our differential equation: $y = A\cos x + B\sin x$

$$y^{I} = -A\sin x + B\cos x$$

$$y^{II} = -A\cos x - B\sin x$$

$$-Acosx - Bsinx + 5(-Asinx + Bcosx) +$$

$$6(Acosx + Bsinx) = sinx$$

$$(5A + 5B)cosx + (5B - 5A)sinx = sinx$$

so if we subject A and B to condition (c):

$$y = -\frac{\cos x}{10} + \frac{\sin x}{10}$$

will be, it can be seen that the general solution of our given differential equation is: $y = c_1 e^{-2x} + c_2 e^{-3x} - \frac{\cos x}{10} + \frac{\sin x}{10}$

will be.[1]

RESULTS

1. Since the degree of the characteristic polynomial is n, the number of fundamental solutions is also n.
2. The solution space is an n-dimensional linear space.

3. If the roots are different, the solutions consist of linearly independent exponential functions.

4. If there is a multiple root, the solution includes x^k powers as multipliers.

5. In the presence of complex roots, oscillating solutions are formed.[1]

High-order linear differential equations with invariant coefficients are theoretically and practically important mathematical models. Their main feature is that the problem is reduced to the solution of an algebraic characteristic equation. The characteristic polynomial roots completely define the structure and behavior of the solution of the equation.

As a result of the analysis, the following was determined: If the order of the equation is n, then the number of fundamental solutions is also n; The space of solutions is a linear space, and the principle of superposition is fulfilled.

Real roots represent exponentially growing or decaying processes. Complex roots create oscillatory processes. When multiple roots are present, the structure of the solution includes multiple multipliers.

In non-homogeneous equations, the general solution consists of a sum of homogeneous and specific solutions.

Thus, high-order differential equations with constant coefficients have a complete theoretical basis, and their solution is carried out by a precise algorithmic process.

Suggestions. In the study of higher-order equations, it is recommended to use numerical methods to determine the roots of the characteristic equation more widely.

In practical issues (mechanics, electrical engineering, vibration theory), it is necessary to pay special attention to stability analysis during the modeling process.

It is useful to use comparative analysis methods when moving from equations with constant coefficients to equations with variable coefficients.

It is recommended to study the theoretical significance of the Wronsky determinant in determining the system of fundamental solutions.[1]

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