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MODELING THE STRESS–STRAIN STATE OF WELLS SUBJECTED TO SEISMIC WAVE EFFECTS

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Abstract. This study investigates the stress–strain state of wells subjected to seismic wave propagation. The work is grounded in the fundamental principles of seismic exploration, employing both reflection and refraction methods to evaluate the elastic and dynamic properties of geological formations. A mechanical model of the wellbore region is constructed to analyze the interaction between seismic waves and the surrounding soil–rock medium. The research further examines the propagation behavior, deformation characteristics, and energy distribution associated with seismic disturbances. Special attention is given to the instrumentation, data acquisition, and interpretation techniques essential for high-accuracy measurements. The findings contribute to advancing geotechnical assessments, improving well design and drilling efficiency, and reducing the risks associated with seismic impacts on subsurface structures

Introduction

Hydraulic engineering investigates the influence of water flow on hydraulic structures and riverbeds, as well as methods for protecting them from the erosive and destructive effects of water. It also develops techniques for regulating river discharge and controlling sediment transport [1–3].Globally, various theoretical and computational models have been developed to account for the interaction between deformable soil environments and hydraulic structures such as wells. Based on these models, specialized methods and computer programs are created to address specific engineering problems, which represent an important direction of targeted scientific research. In this context, during the design of fluid wells, it is essential to understand the physical processes occurring within elastic or viscoelastic half-spaces, the stress fields generated around the wellbore, and the contact interactions between the soil medium and the casing.

A review of the literature and the adopted research methodology shows that the study focuses on wave propagation along the well axis, including the casing, the metallic column, and the surrounding medium, analyzed through various mechanical and mathematical models [4–5].

Moreover, considering the physical and mechanical properties of both the well and the soil medium, the development of reliable analytical and numerical methods for determining the stress–strain state of well casings remains an urgent and practically significant problem [6–7].In this work, we address the problem of wave propagation in a cylindrical layer embedded within an infinite medium (Figure 1) and the resulting stress–strain state. Mathematical modeling is employed to describe these coupled phenomena [8–9].

The relationship between stress and strain in a viscoelastic medium (Hooke's law) is expressed through the hereditary Boltzmann–Volterra integral. The equation of motion for the medium is formulated using Lame's equations within the framework of viscosity theory. The boundary-value formulation leads to a mixed problem in the theory of elasticity. To obtain the solution, several advanced techniques of mathematical physics were applied, including the method of variable separation, special functions, Laplace transforms, Müller's method, and freezing techniques. Numerical results were obtained and analyzed to illustrate the physical behavior of the system.

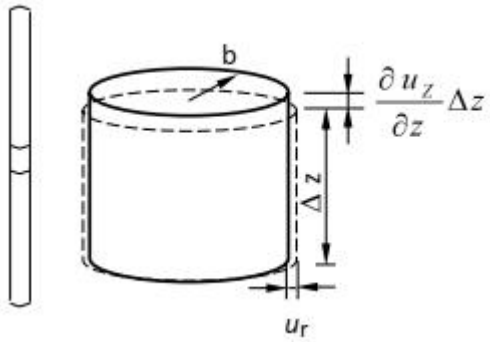


Figure1. Elemental volume change of fluid in the well (u_r and u_z, respectively, radial and axial displacements).

The analysis of the obtained results was carried out in detail. Within the framework of the dispersion relations, it was established that, under certain parameter conditions, a non-dispersive wave can exist on the free surface of the medium. In addition to it, the free vibrations of a cylindrical cavity embedded in an elastic medium were investigated. In this formulation, the problem is reduced to a plane strain state, allowing analytical simplification. The following designations are introduced: denotes the displacement vector of points located on the surface of the cylindrical cavity, while and represent the unit normal and tangential vectors, respectively.

For the longitudinal and transverse potentials, the corresponding displacement components are expressed as follows [10]:

$$u_r=\frac{\partial\varphi}{\partial r}+\frac{1}{r}\frac{\partial\psi}{\partial\theta}\quad ; u_\theta=\frac{1}{r}\frac{\partial\varphi}{\partial\theta}-\frac{\partial\psi}{\partial r}$$

Accordingly, the wave potential equations reduce to the following form.

$$\nabla^2\varphi=\frac{1}{C_\alpha^2}\frac{\partial^2\varphi}{\partial t^2}\qquad \nabla^2\psi=\frac{1}{C_\beta^2}\frac{\partial^2\psi}{\partial t^2}$$

Accordingly, the solution to the wave equation can be written as follows

$$\begin{pmatrix}\varphi\\ \psi\end{pmatrix}=\sum_{n=0}^{\infty}\begin{pmatrix}\varphi_n(r)\\ \psi_n(r)\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}e^{-i\omega t}\qquad (1)$$

where ω is the angular frequency and t is time

Based on equation (1), the solution to the Helmholtz equation is represented in the following form:

$$\begin{pmatrix}\varphi\\ \psi\end{pmatrix}=\sum_{n=0}^{\infty}\begin{pmatrix}A_nH_n^{(1)}(ar)+B_nH_n^{(2)}(ar)\\ C_nH_n^{(1)}(\beta r)+D_nH_n^{(2)}(\beta r)\end{pmatrix} \quad (2)$$

Here, $H_n^{(1)(2)}(z)$- Hankel functions of the first and second kind of order n, $\alpha=\frac{\omega}{C_p}$ va $\beta=\frac{\omega}{C_s}$ – wave number; A_n, B_n, C_n, D_n- integration constants.

In a medium consisting of k cylindrical shell layers, the displacements are expressed in terms of Bessel and Neumann functions of order n with complex arguments.

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$$u_{rk}=\sum_{n=0}^{\infty}\left\{\gamma_N[A_{1NK}\,J_n(\gamma_{1N}r)+A_{2NK}\,Y_n(\gamma_{1K}r)]+\frac{n}{r}[A_{3NK}\,J_n(\gamma_{1N}r)+A_{2NK}\,Y_n(\gamma_{1K}r)]-\frac{r\alpha\gamma_{2k}}{\mu_{2k}}[A_{5NK}\,J_n'(\gamma_{2N}r)+A_{6NK}\,Y_n'(\gamma_{2j}\right.$$

(3)

$$u_{\varphi k}=-\sum_{n=0}^{\infty}\frac{n}{r}\left\{\gamma_X[A_{1NK}\,J_n(\gamma_{1N}r)+A_{2NK}\,Y_n(\gamma_{1K}r)]+\frac{r\gamma_{2x}}{n}[A_{3NK}\,J_n'(\gamma_{2k}r)+A_{4nk}\,Y_n'(\gamma_{2k}r)]-\frac{r}{\mu_{2k}}[A_{5NK}\,J_n(\gamma_{2N}r)+A_{6kn}\,Y_k(\gamma\right.$$

$$u_{zk}=\sum_{n=0}^{\infty}\left\{\gamma_k[A_{1kn}\,J_n(\gamma_{1k}r)+A_{2kx}\,Y_n(\gamma_{1k}r)]+\frac{\gamma_{2k}^2}{n}[A_{5kn}\,J_n(\gamma_{2k}r)+A_{6kn}\,Y_n(\gamma_{2k}r)]\right\}\cos n\varphi e^{j(-6N+\gamma\bar{p})}$$

$\gamma_{1k}^2=\bar{\mu}_{1k}^2-\gamma^2$; $\bar{\mu}_{1k}=\frac{\omega}{a_{1k}\Gamma_k}$; $\gamma_{2k}^2=\bar{\mu}_{2k}^2-\gamma^2$;

$\bar{\mu}_{2k}=\frac{\omega}{c_{sk}\Gamma_{2k}}$; $\gamma=\frac{m\pi}{l}$, $(m=1,2,\ldots.)$ $(k=1,2,\ldots.)$

Voltages are found as:

$$\sigma_{xn}=\frac{\sigma_{\pi k}/h}{\sigma_0^{(p)}}=\frac{E/\rho C_1}{1-\nu^2}\left[\frac{M^2*\ {}^{2}\!/_{\!\nu}h}{24*\ (M+N_1m)}+\cos\theta^2\frac{M(1-V)+N_1m(1-V^2)}{M+N_1m}-\frac{VMt^2(\sin\theta)^2\cos 2\theta}{M(\tau^2-1)+N_1m(2\tau^2-1)}\right]$$

$$\sigma_N=\frac{\sigma_{\theta\theta}/h}{\sigma_0^{(p)}}=\frac{E/\rho C^2}{1-\nu^2}\left[\frac{Mt^2}{2*\ (M+N_1m)}-(\cos\theta)^2\frac{M(1-V)}{M-mN}-\frac{Mt^2(\sin\theta)^2\cos 2\theta}{M(\tau^2-1)+N_1m(2\tau^2-1)}\right],$$

Here, $M=\frac{\mu}{G}$

To determine the arbitrary constants, the boundary conditions are applied. We consider the free vibrations of a cylindrical cavity located within an infinite elastic medium. For this purpose, the stress-free condition is imposed on the surface of the cavity, that is, at $=a$, where the boundary is assumed to be free of external stresses.

$\sigma_{rr}\left|\sigma_{y-a}=\sigma_{y\theta}\right|_{y-a}=0$.

Under free vibration conditions, the reduced Sommerfeld radiation condition is introduced at infinity to describe the attenuation of outgoing waves and ensure the uniqueness of the solution.

$$\left(\lim_{r\to x}\left(\sqrt{\frac{\partial\phi}{\partial r}}+ia\phi\right)\right)^{\sqrt{y}}=0\,,\qquad\left(\lim_{r\to x}\left(\sqrt{\frac{\partial\psi}{\partial r}}+i\beta\phi\psi\right)\right)^{\sqrt{y}}=0$$

By applying the boundary conditions to equation (2), the corresponding frequency equation is obtained, which consists of a combination of Hankel functions. Consequently, determining the arbitrary constants leads to a homogeneous system of algebraic equations. The condition for the existence of a non-trivial solution requires that the determinant of the coefficient matrix vanish.

$\Delta(C,\omega,\gamma_p,\lambda)=0$ (4)

The variation of the complex phase velocity $C=C_R+iC_j$ and the complex group velocity $V=V_R+iV_j$ with respect to the wave number γ_p is determined using the Müller method.If the cylindrical layer is located within an infinite medium, the numerical results were obtained for the following input parameters:

$V_1=0,2$; $V_0=0,25$; $\rho=\frac{\rho_1}{\rho_0}=1,10$; $\mu=\frac{\mu_1}{\mu_2}=0,31$; $\gamma=\frac{c_{p1}}{c_{p2}}=0,7$

Considering the condition for the existence of a non-trivial solution to the homogeneous algebraic system with complex coefficients, the frequency equation can be expressed in the following form:

$$D_n=xH_{n-1}\left[(n^2-1)yH_{n-1}(y)-\frac{n^3+n^2-n}{2}H_n(y)\right]-H_n(x)\left[\frac{n^3+n^2-n}{2}yH_{n-1}(y)-\frac{n^2-n^2+n}{4}y^2H_n(y)\right]=0 \quad (5)$$

Here $=\omega a\left(\frac{\rho}{\lambda+2\mu}\right)^{\frac{1}{2}}$; $y=\omega a\left(\frac{\rho}{\mu}\right)^{\frac{1}{2}}$, λ and μ – Lame koeffitsiyenti

In this expression, ρ represents the density of the material. Upon performing a few simple transformations, equation (5) can be rewritten as follows:

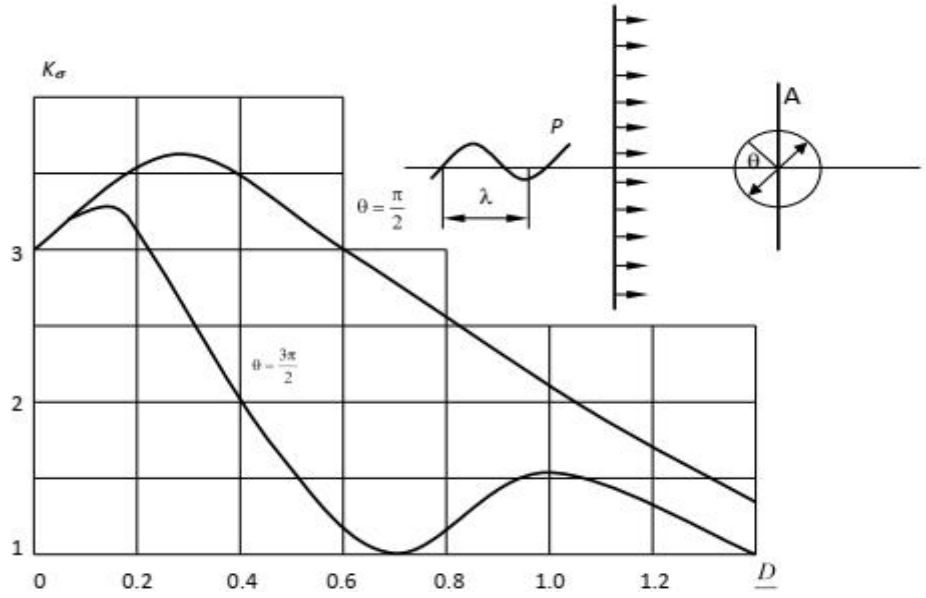


Figure 2. The change of the dynamic power concentration coefficient with respect to the wavelength

When the wavelength tends to infinity and the rheological parameters approach zero, the maximum circumferential (hoop) stress is expressed as follows:

$$\sigma_{\theta\theta}=\frac{2Gv_0}{c_s}\left(1-\frac{c_s^2}{c_\rho^2}\right)\sin 2\theta \sin \omega t,$$

where G is the shear modulus of the soil, and is the amplitude of the incident seismic wave.

Since the wavelength of seismic waves is significantly greater than the transverse dimensions of the pipe, the condition $\frac{D}{\lambda}<1$ holds.The dynamic stress concentration coefficient, , is defined as the ratio of the maximum circumferential stress to the amplitude of the incident wave.

Conclusion

This study examines the stress–strain state of well structures under seismic wave effects in a viscoelastic medium using mathematical modeling and wave propagation theory. The interaction between seismic waves and the wellbore system was analyzed considering both elastic and dissipative properties of the surrounding soil–rock environment.

The results show that stress concentration strongly depends on the ratio of well diameter to wavelength. In the long-wave region (D/λ=0.04–0.16) , the dynamic stress concentration coefficient exceeds the static value by 5–10%, indicating amplification of stresses under low-frequency seismic loading. For D/λ>0.16, the dynamic coefficient becomes smaller than the static one, suggesting reduced stress concentration at higher frequencies.

Accounting for viscous properties decreases the dynamic stress concentration by 10–15%, highlighting the importance of energy dissipation. Differences between ideal elastic and viscoelastic models reach up to 15% in the long-wave region and up to 40% in the short-wave region, emphasizing the need for realistic material modeling.

Overall, the findings improve the understanding of wave-induced stresses in wells and can be applied to enhance design reliability and safety in seismic conditions. Future research should consider more complex media and experimental validation to further increase model accuracy.

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