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NUMERICAL-ANALYTICAL STUDY OF THE PROBLEM OF HEAT CONDUCTIVITY

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Abstract. In this paper, the method of undetermined coefficients is used, where the finite difference scheme is constructed as a linear combination of the values of an unknown grid function at the template's node points. The coefficients of this linear combination are determined by the condition of approximating the given differential equation as accurately as possible across the mesh layers.

Keywords: method of coefficients, grid function, finite difference scheme, differential equation, approximation, Cauchy problem.

Kirish. In the method of unknown coefficients, a linear combination of the values of the unknown mesh function at the nodes forming the template is taken as the differential scheme. The coefficients of this linear combination are found by the condition that the differential scheme approximates the given differential equation to the highest possible order over the mesh layers. [2].

Example,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for the equation

$$U(x,t) = \left\{ \left(x_{i-1}, t_j\right); \left(x_i, t_j\right); \left(x_{i+1}, t_j\right); \left(x_i, t_{j+1}\right); \right\}$$

let the template require the construction of a separate schema. So the separate schema

$$\alpha u_{i-1}^{j} + \beta u_i^{j} + \gamma u_{i+1}^{j} + \mu u_i^{j+1} = 0. \qquad (1)$$

it seems. u_{i-1}^{j} , u_{i+1}^{j} and u_i^{j+1} network functions (x_i, t_j) Taylor series expansion around the point, (1) let's put it into the equation.

$$\alpha u_{i-1}^{j} + \beta u_i^{j} + \gamma u_{i+1}^{j} + \mu u_i^{j+1} =$$

$$(\alpha + \beta + \gamma + \mu) u_i^{j} + (\gamma - \alpha) h \frac{\partial u\left(x_i, t_j\right)}{\partial x} + \frac{h^2}{2}(\alpha + \gamma) \frac{\partial^2 u\left(x_i, t_j\right)}{\partial x^2}$$

$$+\mu\tau\frac{\partial u(x_i,t_j)}{\partial t}+O\left(h^3+\tau^2\right). \qquad (2)$$

(2) from equality $\alpha u_{i-1}^j+\beta u_i^j+\gamma u_{i+1}^j+\mu u_i^{j+1}=(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2})_{(x_i,t_j)}+O(h^3+\tau^2)$ to be α,β,γ and μ the coefficients must satisfy the following system of equations.

$$\begin{aligned}&\alpha+\beta+\gamma+\mu=0\\&\gamma-\alpha=0\\&\alpha+\gamma=-\frac{2}{h^2}\\&\mu\tau=1\end{aligned} \qquad (3)$$

(3) solve the system of equations, $\alpha=\gamma=-\frac{1}{h^2}$, $\mu=\frac{1}{\tau}$ and $\beta=\frac{2}{h^2}=-\frac{1}{\tau}$ we determine that. These values of the coefficients

$$\frac{d}{d\tau}L_1w+\ k_1\phi_1(\tau)-\frac{p-1}{m(p-1)-1}\ L_1w+k_2L_2w-k_3w\phi_2(\tau)-k_4w^q\phi_2(\tau)=0$$

we put it on:

$$\frac{u_i^{j+1}-u_i^j}{\tau}-\frac{u_{i-1}^j-2u_i^j+u_{i+1}^j}{h^2}=0 \qquad (4)$$

Ergo,

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

differential equation $U(x_i,t_i)=\{(x_{i-1},t_j);(x_i,t_j);(x_{i+1},t_j);(x_i,t_{j+1});\}$ The approximate differential scheme in the template looks like this:

$$\frac{u_i^{j+1}-u_i^j}{\tau}=\frac{u_{i-1}^j-2u_i^j+u_{i+1}^j}{h^2}.$$

Now the $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ differential equation

$$U(x_i,t_i)=\{(x_{i-1},t_j);(x_i,t_j);(x_{i+1},t_j);(x_i,t_{j+1});\}$$

we find the approximate differential equation in the template. In this case, the differential scheme

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=0. \qquad (5)$$

will be visible.

$$\frac{d}{d\xi}\left|\frac{d\varphi^m}{d\xi}\right|^{p-2}\frac{d\varphi^m}{d\xi}+\gamma\xi\frac{d\varphi}{d\xi}-\alpha\varphi=0$$

in u_{i-1}^{j+1}, u_i^{j+1}, u_{i+1}^{j+1} mesh functions (x_i,t_j) around a point Expanding into a Taylor series, we get:

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=[\alpha+\beta+\gamma+\mu]u(x_i,t_j)+h(\mu-\beta)u_x(x_i,t_j)+$$
$$+\tau(\beta+\gamma+\mu)u_t(x_i,t_j)+0{,}5h^2(\beta+\mu)u_{xx}(x_i,t_j)+0{,}5\tau^2(\beta+\gamma+\mu)u_{tt}(x_i,t_j)+$$
$$+h\tau(\mu-\beta)u_{xt}(x_i,t_j)+\frac{h^3}{6}(\mu-\beta)u_{xxx}(x_i,t_j)+\frac{\tau^3}{6}(\beta+\gamma+\mu)u_{ttt}(x_i,t_j)+$$
$$+0{,}5h^2\tau(\beta+\mu)u_{xxt}(x_i,t_j)+0{,}5h^2\tau(\mu-\beta)u_{xtt}(x_i,t_j)+\frac{h^4}{24}(\beta+\mu)u_{xxxx}+$$
$$+\frac{h^3\tau}{6}(\mu-\beta)u_{xxxt}(x_i,t_j)+\frac{h^2\tau^2}{4}(\beta+\mu)u_{xxxt}(x_i,t_j)+\frac{h\tau^3}{6}(\mu-\beta)u_{xttt}(x_i,t_j)+$$
$$+\frac{h\tau^3}{6}(\mu-\beta)u_{xttt}(x_i,t_j)+\frac{\tau^4}{24}(\beta+\mu)u_{tttt}(x_i,t_j)+....$$

In the last equation, we require that the following be true:

$$\alpha+\beta+\gamma+\mu=0$$
$$\beta+\gamma+\mu=\frac{1}{\tau}$$
$$\beta+\mu=-\frac{2}{h^2}$$
$$\mu-\beta=0$$

Solving this system of equations, we find the unknown $\alpha, \beta, \gamma, \mu$ we define the values of the parameters:

$$\alpha = -\frac{1}{\tau}, \quad \beta = \mu = -\frac{1}{h^2}, \quad \gamma = \frac{1}{\tau} + \frac{1}{h^2}.$$

At these values of the parameters

$$\alpha u_i^j + \beta u_{i-1}^{j+1} + \gamma u_i^{j+1} + \mu u_{i+1}^{j+1} = \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2}_{(x_i, t_j)} - 0{,}5\tau u_{tt}(x_i, t_j) + \frac{\tau^2}{6} u_{ttt}(x_i, t_j) -$$

$$-\frac{h^2}{12} u_{xxxx}(x_i, t_j) - \frac{\tau^2}{2} u_{xxxx}(x_i, t_j) + -\frac{h^2\tau^4}{12} u_{tttt}(x_i, t_j) + \ldots . \qquad 5$$

$\alpha, \beta, \gamma, \mu$ If we put the found values of the parameters into equation (5), we get the form of the differential circuit we built::

$$\frac{u_i^{j+1} - u_i^j}{\tau} - \frac{u_{i-1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}}{h^2} = \frac{\partial u(x_i, t_j)}{\partial t} - \frac{\partial^2 u(x_i, t_j)}{\partial t^2} + O(\tau, h^2).$$

Ergo, $U(x_i, t_i) = \{(x_{i-1}, t_j); (x_i, t_j); (x_{i+1}, t_j); (x_i, t_{j+1});\}$ The differential scheme built on the template is not transparent, and the given differential problem is τ with the first order and h while approximating with the second order.

Conclusion. Numerical schemes for nonlinear boundary value problems are covered. The first part covers differential equations, examples of differential equations, and differential approximation methods, and an analysis of numerical solutions is presented.

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