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CALCULATION OF PARAMETER-DEPENDENT INTEGRALS IN THE MATHEMATICA SYSTEM

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Abstract. This article analyzes the possibilities of analytical and numerical calculation of integrals depending on a parameter in the computer mathematics system Mathematica. Using Mathematica, issues such as differentiating integrals, passing to the limit, and imposing restrictions on parameters are illustrated through examples. The codes and explanations provided in the article serve as a practical guide for students and teachers.

Keywords: integral, integral with respect to a parameter, Mathematica, Wolfram, symbolic calculus, Leibniz's rule, improper integral.

Introduction. The calculation of integrals depending on a parameter is an important and sometimes difficult part of mathematical analysis. Manual calculation of such integrals is time-consuming and can lead to errors. The problem becomes even more difficult if the function under integration is complex or if the approximation of the integral changes for different values of the parameter.

Mathematica is one of the most powerful computer mathematics systems developed by Wolfram Research. Like Maple, it has the capabilities of symbolic computation, numerical integration, graphical visualization, and solving complex mathematical problems. Mathematica has the following advantages:

Powerful symbolic integration engine
Easy to insert constraints on parameters (Assumptions)
Automatically check improper integrals
Beautiful and professional results (prepared in TeX, PDF, HTML formats).

This article will illustrate the capabilities of the Mathematica system in calculating integrals depending on the parameter with examples.

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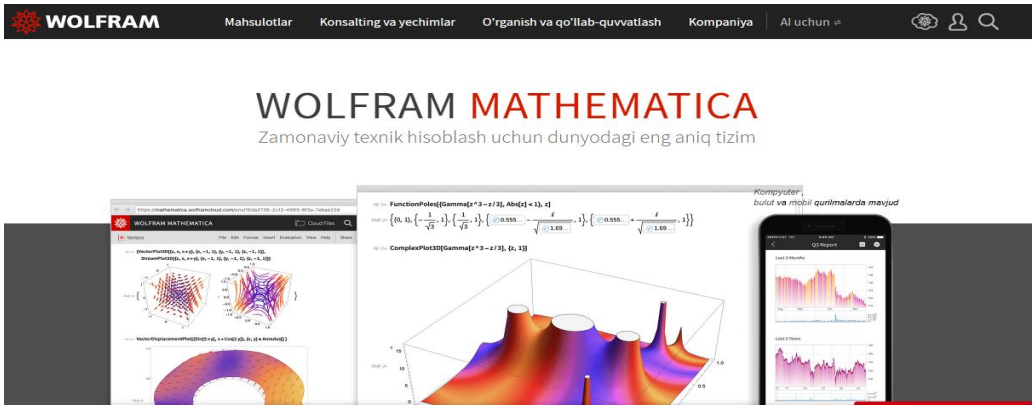


Figure 1. General view of the Mathematica system.

1. The concept of integral with respect to a parameter

If the function f(x,α) is integrable on the interval [a;b] for each parameter value α, then

$$I(\alpha)=\int ab \int_a^b f(x,\alpha)dx$$

the integral is called a parameter-dependent integral. If the limits of integration also depend on the parameter:

$$\Phi(\alpha)= \int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)dx.$$

Such integrals may be unique or unique at different α. Therefore, a separate check is required in each case.

2. Basic theorems

2.1. Go to the limit

If f(x,α) is continuous in a rectangle, then:

$$\lim_{\alpha\to\alpha_0} \int_a^b f(x,\alpha)dx = \int_a^b \lim_{\alpha\to\alpha_0} f(x,\alpha)dx\,.$$

2.2. Differentiation (Leibniz's rule)

If $f(x,\alpha)$ and $\frac{\partial f}{\partial \alpha}$ continuous:

$$I'(\alpha)= \int_a^b \frac{\partial f(x,\alpha)}{\partial \alpha} \quad dx.$$

If the limits depend on the parameter:

$$\Phi'(\alpha) = f(\psi(\alpha);\alpha)\psi'(\alpha) - f(\varphi(\alpha);\alpha)\varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial f}{\partial \alpha}dx$$

2.3. Integrating with respect to a parameter

$$\int_{\alpha_1}^{\alpha_2} \left(\int_a^b f(x,\alpha)dx \right) d\alpha = \int_a^b \left(\int_{\alpha_1}^{\alpha_2} f(x,\alpha)d\alpha \right) dx$$

3. Indefinite integrals and their approximation

$$I(\alpha) = \int_a^{+\infty} f(x,\alpha)dx$$

This is called a proper integral. It is said to converge on $\alpha \in [\alpha 1;\alpha 2]$ if $\lim_{B \to \infty} \int_a^B f(x,\alpha)dx$ exists and is finite. Absolute approximation is $\int_a^{+\infty} |f(x,\alpha)|\, dx$ is the convergence of.

Smooth convergence condition: for any ε>0, find A such that for all B,C>A and for all α:

$$\left| \int_C^B f(x,\alpha)dx \right| < \varepsilon.$$

These conditions can be checked using Mathematica.

4. Calculating integrals in Mathematica

The functions **Integrate[]** and **NIntegrate[]** are mainly used to calculate integrals in Mathematica.:

Integrate[*f*[*x*], {*x*, *a*, *b*}] – calculates the definite integral symbolically
NIntegrate[*f*[*x*], {*x*, *a*, *b*}] – calculates the definite integral numerically
D[*f*, *var*] – differentiation
Limit[*f*, *var* –> *value*] – go to the limit
Assuming[*cond*, *expr*] – impose restrictions on parameters

4.1. Simple example (Leibniz's rule)

Problem:

.

$$I(\alpha) = \int_1^2 \frac{e^{\alpha x^2}}{x} dx.\ I'(\alpha)$$

Mathematica code:

```
mathematica

(* Integralni hisoblash
g = Integrate[Exp[a*x^2

(* Hosilani topish *)
D[g, a]
```

Result:

$$\frac{e^{4a} - e^{a}}{2a}$$

Analysis: Mathematica expressed the integral in a compact form, rather than using special functions (ExpIntegralEi) as in Maple. This result is consistent with the result calculated by hand. Unlike Maple, Mathematica often tries to express the results in terms of elementary functions..

4.2. Go to the limit

Problem:

$$\lim_{\alpha\to 0}\int_{-\pi}^{\pi}(x+\cos(\alpha x))e^{x\sin\alpha}dx$$

Mathematica code:

```
mathematica

g = Integrate[x + Cos[a*x]*Exp[x*Sin[a]], {x, -Pi, Pi}]
Limit[g, a -> 0]
```

Result: 2π.

Analysis: Mathematica is faster and more direct than Maple in calculating limits. While this example would have produced a complex expression in Maple, Mathematica produces a more compact result.

4.3. Limiting a parameter (indefinite integral)

Problem:

$$\int_0^{+\infty} e^{-\alpha x}dx$$

If no constraint is imposed, Mathematica will give a conditional answer.:

```
mathematica

Integrate[Exp[-a*x], {x, 0, Infinity}]
```

Result:

$$ConditionalExpression[1/a, Re[a] > 0]$$

With a limit:

```
mathematica

Assuming[a > 0, Integrate[Exp[-a*x], {x, 0, Infinity}]]
```

Result:

$\frac{1}{a}$.

Analysis: Unlike Maple, Mathematica also provides conditional answers without constraints (ConditionalExpression). This is a very convenient feature, as it automatically shows at what values of the parameter the integral converges.

4.4. Integral whose limits depend on a parameter

Problem:

$$g(\alpha)=\int_{\alpha}^{2\alpha}\frac{\sin(\alpha x)}{x}dx,\quad g'(\alpha)=?$$

Mathematica code:

```
mathematica

g = Integrate[Sin[a*x]/x, {x, a, 2*a}]
D[g, a]
```

Result:

$$-((Sin[a\text{^}2] - Sin[2\ a\text{^}2])/a) + (SinIntegral[2\ a\text{^}2] - SinIntegral[a\text{^}2])/a$$

Analysis: The result is the same as in Maple, but Mathematica uses the SinIntegral function (Si in Maple). Both programs produce the same mathematical result.

Conclusion. Mathematica is a powerful and convenient tool for calculating integrals with respect to parameters. It provides the following capabilities:

Analytical and numerical calculations (Integrate, NIntegrate);
Differentiation and taking limits (D, Limit);
Automatic checking of improper integrals (ConditionalExpression);
Setting constraints on parameters (Assuming);
Interactive visualization (Manipulate, Plot).

Compared to Maple, Mathematica is more convenient in automatically determining parameter conditions with its ConditionalExpression feature. Also, its graphical and interactive capabilities make the learning process more interesting for students.

Based on the examples and codes provided in the article, students and teachers can independently master this topic. With the help of Mathematica, complex problems in mathematical analysis can be solved quickly and reliably.

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