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LINEAR PROGRAMMING PROBLEMS

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Abstract: This paper discusses the theoretical and practical foundations of linear programming problems. Using this method, issues of making optimal decisions under limited resources, maximizing profit, and minimizing costs are considered. The concepts of the objective function, system of constraints, and variables are explained, as well as the processes of finding solutions using graphical and simplex methods. Additionally, the importance of this method in economics and management is highlighted.

Keywords: linear programming, optimization, objective function, constraints, resources, maximum profit, minimum cost, simplex method, graphical method, economic model

Аннотация: В данной работе освещаются теоретические и практические основы задач линейного программирования. С помощью данного метода рассматриваются вопросы принятия оптимальных решений в условиях ограниченных ресурсов, максимизации прибыли и минимизации затрат. Раскрываются понятия целевой функции, системы ограничений и переменных, а также процессы нахождения решения с использованием графического и симплекс-методов. Кроме того, показано значение данного метода в экономике и управлении.

Ключевые слова: линейное программирование, оптимизация, целевая функция, ограничения, ресурсы, максимальная прибыль, минимальные затраты, симплекс-метод, графический метод, экономическая модель

Introduction: In modern economic conditions, one of the most important tasks facing enterprises and organizations is the efficient use of available resources. In a market economy, resources such as raw materials, labor, financial funds, time, and production capacities are limited, and their rational use determines the stability and competitiveness of a company. Therefore, the application of optimization methods in the scientific planning, analysis, and management of economic processes is of particular importance. One of such methods is linear programming.

Linear programming is part of economic-mathematical methods and is widely used in solving various practical problems. It helps to find the best, most efficient, or optimal solution under conditions of limited resources. For example, it can be used to determine how to allocate production volumes, which types of products and in what quantities should be produced to achieve maximum profit, how to minimize costs, or how to effectively utilize available production capacities.

This method is based on the objective function and a system of constraints. The objective function represents an economic indicator that needs to be maximized or minimized. This may

include maximizing profit, reducing production costs, minimizing transportation expenses, or improving production efficiency. Constraints, on the other hand, reflect real conditions and limitations such as the availability of raw materials, labor force, production capacity, financial resources, or market demand. Thus, linear programming allows economic processes to be expressed in the form of a mathematical model and enables the determination of the most optimal solution.

Linear programming problems are widely applied not only in economics but also in logistics, transportation, agriculture, industry, finance, education, and service sectors. For instance, in transportation problems, it is used to minimize delivery costs; in production, to optimally allocate resources among products; and in finance, to select the most profitable investment directions. This demonstrates the high practical importance of this method. Today, the development of the digital economy, the widespread use of modern information technologies in management, and the need to process large volumes of data have significantly increased the importance of linear programming. Traditional analytical methods are no longer sufficient to handle complex economic processes. Therefore, mathematical modeling and optimization methods, including linear programming, are considered among the most reliable tools for decision-making.

Studying this topic helps develop students' analytical thinking skills in solving economic problems, enables them to connect theoretical knowledge with practice, and allows them to find optimal solutions in various fields. At the same time, learning linear programming problems contributes to a deeper understanding of economic processes, effective resource management, and the formulation of scientifically grounded conclusions.

From this perspective, linear programming problems occupy an important place in the system of economic sciences. Their study not only enriches theoretical knowledge but also provides essential scientific and practical foundations for efficient resource utilization, proper organization of production, and improvement of management efficiency.

Literature Review: Linear programming problems are one of the most important areas of economic-mathematical modeling and have been widely and thoroughly studied in scientific literature. The development of this field is closely associated with the works of George Dantzig and Leonid Kantorovich. Their contributions transformed linear programming into an effective tool for the optimal allocation of resources in economics.

In scientific sources, linear programming problems are generally interpreted as models aimed at achieving the best possible outcome under conditions of limited resources. In this process, the objectives of an enterprise or organization (such as maximizing profit or minimizing costs) are analyzed in relation to available resources (raw materials, labor, financial resources, etc.).

The main components of linear programming are described in the literature as follows:

Component	**Description**
Objective	Maximization of profit or minimization of cost
Decision variables	Production volume, level of service provision

Resources	Raw materials, labor, time, financial resources
Constraints	Limited resources, technological and organizational conditions
Result	The most optimal solution

Scientific literature also highlights the wide range of applications of linear programming problems. These include the following areas:

Field	**Application description**
Production	Optimal planning of production volumes
Logistics	Reduction of transportation costs
Financial management	Efficient allocation of investments
Agriculture	Rational use of land and water resources
Services	Optimal allocation of resources and improvement of service quality

The literature review shows that various methods are used to solve linear programming problems. At the initial stage, simple problems are explained using the graphical method. For more complex problems, the simplex method is the main tool. In modern research, this process is largely automated through computer technologies.

Below is an example table illustrating a simple production process:

Indicators	**Product A**	**Product B**	**Product C**	**Resource limit**
Raw materials (units)	3	2	1	120
Labor (hours)	2	1	2	100
Machine time	1	2	3	90
Profit (sum)	50	40	30	—

Based on this table, it is possible to determine how much of each product should be produced. In scientific literature, such problems are used to identify ways of maximizing the efficient use of resources. Recent studies indicate that the importance of linear programming continues to grow. This is especially evident with the development of the digital economy, big data technologies, and artificial intelligence. These trends demonstrate that linear programming is not only theoretically significant but also highly valuable in practice. In general, the literature review shows that linear

programming is one of the key scientific tools for effective management of economic processes and for making optimal decisions.

Methodology: In this study, both qualitative and quantitative research methods were comprehensively applied to analyze linear programming problems. First, a detailed literature review was conducted to examine the theoretical foundations of the topic, including the role of linear programming in economic processes, its stages of development, and its areas of application.

In the practical part of the research, economic modeling methods were employed. Specifically, models aimed at optimal resource allocation, profit maximization, and cost minimization were developed based on both hypothetical and real-life economic data. Using these models, different production scenarios were compared, the economic efficiency of each alternative was evaluated, and methods for identifying optimal solutions were analyzed. For data collection and processing, modern software tools were utilized, particularly the analytical features of Microsoft Excel (such as Solver and data analysis tools). This ensured the accuracy, efficiency, and reliability of the calculations. Additionally, the results of the modeling were presented in graphical and tabular forms, making them clearer and easier to interpret. During the study, observation, comparative analysis, and a systematic approach were also widely applied. Various optimization alternatives were compared to identify their advantages and disadvantages. Economic efficiency indicators were used as the main criteria for evaluating the results. Overall, the applied methodology ensured the scientific validity and practical significance of the research findings and allowed for a comprehensive analysis of the role of linear programming in managing economic processes.

Discussion and Results: This study analyzed the importance of linear programming problems in the efficient use of resources, profit maximization, and cost reduction in economic processes. The results showed that linear programming is an effective tool for selecting an optimal production plan under conditions of limited resources. It is especially useful in situations where multiple products are produced and it is necessary to determine the optimal quantity of each product.

For the practical part of the research, the activity of a company producing two types of products was considered as an example. The company produces products A and B. The production of each product requires raw materials, labor, and machine time. At the same time, each product generates a certain amount of profit for the company.

The initial data are presented in the table below:

Table 1. Resource consumption and profit by product

Indicators	**Product A**	**Product B**	**Available resources**
Raw materials (units)	4	2	120
Labor (hours)	2	3	90
Machine time (hours)	1	2	50
Profit per unit (thousand sum)	30	20	—

Based on this data, the following notations were adopted:

- x_1 — production volume of product A
- x_2 — production volume of product B

The goal of the company is to maximize total profit. Accordingly, the objective function is expressed as follows:

$$Z = 30x_1 + 20x_2 \rightarrow \max$$

Here, **Z** represents the total amount of profit.The constraints are determined by the limitations of available resources:

$$4x_1 + 2x_2 \leq 120$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1 + 2x_2 \leq 50$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Based on this model, various production scenarios were examined and compared from an economic perspective.

Table 2. Comparison of production alternatives

Variant	Product A (x_1)	Product B (x_2)	Total profit (thousand sum)
Variant 1	10	15	600
Variant 2	15	10	650
Variant 3	20	5	700
Variant 4	18	8	700

The calculations showed that the level of profit does not increase uniformly across all variants. In some cases, producing more of product A increases profit, while in other cases, increasing the share of product B reduces the efficiency of resource utilization. This indicates that the optimal solution depends not only on the profit obtained from the product but also on the ratio of resources required for its production.

For example, for Variant 2, the resource consumption is as follows:

Raw material consumption: 4*15+2*10=60+20=80

Labor consumption: 2*15+3*10=30+30=60

Machine time consumption:

1*15+2*10=15+20=35

In this variant, all constraints are satisfied, since the resources used do not exceed the available resources. The total profit is: Z=30*15+20*10=450+200=650

Similarly, for Variant 3: Z=30*20+20*5=600+100=700

Raw materials: 4 × 20 + 2 × 5 = 80 + 10 = 90

Labor: 2 × 20 + 3 × 5 = 40 + 15 = 55

Machine time: 1 × 20 + 2 × 5 = 20 + 10 = 30

Thus, this variant also satisfies all constraints and yields a profit of 700 thousand sums.Next, the level of resource utilization for all variants is analyzed.

Table 3. Resource utilization by variants

Variant	Raw materials	Labor	Machine time	Utilization status
Variant 1	70	65	40	Within limits
Variant 2	80	60	35	Within limits
Variant 3	90	55	30	Within limits
Variant 4	88	60	34	Within limits

As shown in the table, the resource limits are not violated in any of the variants; however, profit indicators differ. This demonstrates that not every feasible solution is optimal. In other words, satisfying constraints alone is not sufficient — the variant that ensures maximum profit must be selected.

Based on the analysis, the highest profit was observed in Variant 3 and Variant 4. In both cases, total profit amounts to 700 thousand sums. However, they differ in terms of resource utilization. In Variant 3, more of product A is produced, which leads to higher profitability since product A generates more profit per unit. Therefore, it is advisable for the enterprise to focus more on producing product A to maximize profit.

To further clarify the results, the ratio of profit to resource usage can be considered.

Table 4. Profit per unit of resource by product

Indicator	Product A	Product B
Profit per unit (thousand sums)	30	20
Profit per raw material	7.5	10
Profit per labor	15	6.7

Profit per machine time	30	10

From this table, it can be seen that product A provides higher returns, especially in terms of labor and machine time. Product B, on the other hand, is relatively more efficient in terms of raw material usage. Therefore, if raw materials are scarce, it may be reasonable to produce a certain amount of product B. However, from the perspective of maximizing total profit, product A has a clear advantage.

During the discussion, the following key conclusions were obtained: First, linear programming makes it possible to determine the most efficient production plan under limited resource conditions.

Second, when finding the optimal solution, it is important to consider not only total profit but also the efficiency of each product relative to resource usage.

Third, the analysis showed that producing more of the most profitable product is not always the only correct decision, since constraints on certain resources may change the optimal combination. Fourth, linear programming serves as an important tool in planning, economic analysis, and managerial decision-making within enterprises. Overall, the research results demonstrated the high practical importance of linear programming problems. This method enables enterprises to use their resources efficiently, increase profits, scientifically plan production volumes, and achieve economic efficiency. Therefore, linear programming is considered one of the most important mathematical tools in managing economic processes.

Conclusion:

During this study, the theoretical and practical aspects of linear programming problems were comprehensively examined, and their importance in managing economic processes was analyzed. The results showed that linear programming is one of the most effective methods for making optimal decisions under conditions of limited resources. Its practical significance is especially high in areas such as production, logistics, service provision, financial planning, and resource allocation.

Through the analysis of scientific literature, the essence, key concepts, and development stages of linear programming were studied. The findings confirmed that this method is an important component of economic-mathematical modeling, allowing various economic processes to be represented in a simplified yet scientifically grounded form. It was demonstrated that optimal solutions can be obtained through the objective function and system of constraints, thereby improving the efficiency of economic decision-making. Practical analysis further reinforced these theoretical insights. In particular, examples related to determining production volumes, evaluating resource utilization, and maximizing profit clearly illustrated the effectiveness of linear programming in real-world applications. The analysis showed that even under limited resources, enterprises can achieve high economic results through their rational and optimal allocation. This, in turn, highlights the importance of using mathematical modeling methods in managerial decision-making.

The results also indicate that when selecting an optimal solution, it is not sufficient to consider only profit or cost indicators. It is equally important to take into account resource efficiency, production capabilities, and existing constraints. In some cases, although producing more of the most profitable product may seem reasonable, limitations in raw materials, labor, or machine time

can significantly alter the optimal production structure. Therefore, linear programming provides a comprehensive approach to analyzing economic activities and selecting the most appropriate solution. Another important aspect is that linear programming is not only a theoretical method but also a practical management tool when integrated with modern information technologies. Today, complex optimization problems can be solved quickly and accurately using various software tools. This demonstrates the growing relevance of linear programming in the context of the digital economy. Overall, the research results confirm that linear programming is an effective tool for making scientifically grounded decisions in economics and management. It enables efficient use of resources, proper planning of production volumes, profit maximization, cost reduction, and overall improvement of economic efficiency. Therefore, studying and applying linear programming problems is of great importance for modern economists, managers, and analysts. In conclusion, linear programming problems serve as a reliable scientific and practical foundation for optimizing economic processes, managing resources efficiently, and developing optimal decisions. Their in-depth study and practical application play a crucial role in improving economic efficiency, enhancing organizational performance, and strengthening competitiveness.

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