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FIRST-ORDER LINEAR EQUATIONS WITH ONE UNKNOWN AND THEIR APPLICATIONS

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Applications of the concept of derivative in solving geometric problems

Annotatsiya: Tenglamalar orasida eng soddasi birinchi darajali (chiziqli) bir noma'lumli tenglamalar hisoblanadi. Boshlang'ich sinf matematika kursida eng sodda elementar tenglamalar o'rganilib ularni yechish amal komponenti bilan amal natijasi orasidagi bog'lanishni o'qitish metodikasidan foydalaniladi.

Maqolada birinchi darajali bir noma'lumli tenglamalarni yechishga keltiriladigan turli toifadagi masalalar o'rganildi.

Kalit so'zlar: birinchi darajali (chiziqli) bir noma'lumli tenglama, yechim (ildiz), yagona yechim, cheksiz yechim, parametr, ertak va qadimiy masalalar.

Annotation: Among the equations, the simplest are the first-order (linear) single-unknown equations. In the primary grade mathematics course, the simplest elementary equations are studied, the method of solving them is used, teaching the connection between the action component and the result of the action.

The article explores a variety of categories of problems that can be solved by solving first-order unsolved equations.

Keywords: a first-order (linear) single unknown equation, solution (root), single solution, infinite solution, parameter, fairy tale and ancient problems.

Аннотация: Среди уравнений самые простые — это уравнения первого порядка (линейные) с одиночными неизвестными. В курсе математики для начальных классов изучаются самые простые элементарные уравнения, используется метод их решения, обучающий связь между компонентой действия и результатом действия.

В статье рассматриваются различные категории задач, которые можно решить, решая неразрешённые уравнения первого порядка.

Ключевые слова: уравнение первого порядка (линейное) одно неизвестное, решение (корень), единое решение, бесконечное решение, параметр, сказочные и древние задачи.

INTRODUCTION

In the school mathematics curriculum, the concept of a linear equation is introduced through the abstract-deductive method, because first a definition of the equation is given, then its general form, methods of solving it, and its graph are studied.

Definition: If the left and right sides of an equation consist of linear functions with respect to an unknown variable, such an equation is called a first-order (linear) equation.

A linear equation can generally be written in the form:

$ax + b = cx + d$ (1)

Here a, b, c, d are given real numbers and x is the unknown.

We rewrite it as:

$ax - cx = d - b \Rightarrow (a - c)x = d - b$ (2)

If $a - c \neq 0$ ($a \neq c$), then equation (1) has a unique solution:

$x = (d - b) / (a - c)$

If $a - c = 0$ ($a = c$) and $d - b \neq 0$ ($d \neq b$), then equation (2) takes the form:

$0 \cdot x = d - b$

In this case, the left side equals 0 for any value of x, while the right side is not zero. Therefore, the equation has no solution.

If $a - c = 0$ ($a = c$) and $d - b = 0$ ($d = b$), then equation (2) takes the form:

$0 \cdot x = 0$

This equality holds for all values of x, therefore the equation has infinitely many solutions.

Sometimes the equation

$Ax = B$ (3)

is also considered as the general form of a linear equation with one unknown, where A and B are given constants.

Now let us consider applying the theory of solving equation (1) to linear equations with parameters.

Example 1

Solve the equation $m + x = m^2x + 1$ with respect to x, where m is a parameter.

Solution:

$(m^2 - 1)x = m - 1$

$(m - 1)(m + 1)x = m - 1$

If $m \neq \pm 1$, the equation has the unique solution:

$x = 1 / (m + 1)$

If $m = -1$, the equation becomes:

$0 \cdot x = -2$

In this case, the equation has no solution.

If $m = 1$, the equation becomes:

$0 \cdot x = 0$

In this case, the equation has infinitely many solutions, meaning x can be any number.

Example 2

Solve the equation $ax - b = 1 + x$ with respect to x, where a and b are parameters.

Solution:

$(a - 1)x = b + 1$

If $a \neq 1$, then the unique solution is:

$x = (b + 1) / (a - 1)$

If $a = 1$ and $b + 1 \neq 0$ ($b \neq -1$), then the equation becomes:

$0 \cdot x = b + 1$

Thus, the equation has no solution.

If $a = 1$ and $b = -1$, then the equation becomes:

$0 \cdot x = 0$

In this case, the equation has infinitely many solutions.Interesting problems solved using linear equations with one unknown

Problems reducible to linear equations with one unknown were already known about 4000 years ago in Ancient Babylon and Egypt. As evidence, we can cite a problem from the Rhind Papyrus (also known as the Ahmes Papyrus), dated to 2000–1700 BC and preserved in the British Museum:

"If two-thirds of a number is added to the number and then one-third of the resulting sum is subtracted, the result is 10. Find the number."

Solution:

$x + 2/3x - 1/3(x + 2/3x) = 10$

Solving this linear equation gives:

x = 9

A fairy-tale problem

Some fairy-tale problems can also be solved using algebraic methods that lead to linear equations.

A flock of geese was flying in the sky. A single goose flying toward them said:

"Hello, one hundred geese!"

The leader of the flock replied:

"No, we are not one hundred. If as many geese as we are now joined us, then half of that number, then a quarter of that number, and you as well, then we would become one hundred. Now guess how many we are."

The lone goose thought long and hard but could not determine the number.

Although the problem can be solved arithmetically, we will show the algebraic method. Let the number of geese be x.

From the problem:

$x + x + x/2 + x/4 + 1 = 100$

$11x = 396$

$x = 36$

Answer: There were 36 geese in the flock.

A historical poetic problem

The Greek scholar Metrodorus, author of interesting mathematical poems, presented the following famous poetic problem leading to a linear equation:

"Here lies Diophantus, This tomb reveals his age. Through clever calculation, Find how long he lived.

One-sixth of his life was childhood, One-twelfth was youth. Another seventh passed before marriage, Then five years later his son was born.

The son lived half his father's life, Then fate took him away. Four more years Diophantus lived, In grief after his son's death.

Tell me now, clever reader, How many years did Diophantus live?"*

The mathematical model of this poem is:

$1/6x + 1/12x + 1/7x + 5 + 1/2x + 4 = x$

Solving gives:

$x = 84$

Thus, Diophantus lived 84 years.

Note

Diophantus of Alexandria (3rd century) was a mathematician. Algebraic equations with integer coefficients or systems of such equations are called Diophantine equations, and the goal is to find their integer or rational solutions. Such equations usually contain at least two variables.

Diophantine equations often have many solutions, therefore they are also called indeterminate equations. Examples include:

$3x + 5y = 7$

$x^2 + y^2 = z^2$

$3x^3 + 4y^3 = 5z^3$

and others.

Many interesting problems are collected in Diophantus' book "Arithmetica."

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