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SOLVING DIFFERENTIAL EQUATIONS USING NUMERICAL METHODS

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Abstract. This article discusses the theoretical foundations of solving differential equations using numerical methods. Information is provided about the concept of differential equations, their practical significance, and numerical methods used in cases where finding analytical solutions is difficult. The essence, advantages, and application areas of Euler, Runge–Kutta, and Adams methods are analyzed. In addition, the role of numerical methods in modern science and technology is considered.

Keywords. Differential equation, numerical methods, Euler method, Runge–Kutta method, Adams method, approximate solution, computational mathematics, mathematical modeling, algorithm, computer calculations.

Differential equations are one of the important branches of mathematics and are widely used in mathematical modeling of various processes occurring in nature and society. Processes arising in physics, engineering, economics, biology, mechanics, and many other sciences are expressed through differential equations. By solving differential equations, it is possible to determine the future state of a process, study the properties of a system, and analyze various problems.

Finding exact analytical solutions for many differential equations is difficult or impossible. Therefore, numerical methods that allow obtaining approximate solutions play an important role in practical problems. Numerical methods make it possible to solve differential equations using computers and serve as an effective tool for studying complex mathematical models. Nowadays, the development of computing technologies has led to the wider application of numerical methods. In particular, the use of numerical methods is highly relevant in scientific research, technical design, meteorology, economic forecasting, and information technologies.

A differential equation is an equation that contains derivatives of an unknown function. Differential equations are divided into ordinary differential equations and partial differential equations. In ordinary differential equations, the unknown function depends on a single independent variable, whereas in partial differential equations, the unknown function depends on several independent variables. The main purpose of solving differential equations is to determine the unknown function. However, in many cases, it is difficult to find an analytical solution, so approximate methods are used.

Numerical methods are a set of techniques used to find approximate solutions of differential equations. With these methods, solutions are calculated at specific points, and results are obtained through successive approximations. The main idea of numerical methods is to represent a continuous process through discrete points. In this process, the interval is divided into steps, and the values at each point are calculated using special formulas.

Numerical methods have the following characteristics:
provide approximate results;
simplify the calculation process;

are adapted to computer technologies;
make it possible to solve complex differential equations.

The accuracy of numerical methods depends on the step size and the selected method. The smaller the step size, the more accurate the result, but the amount of computation increases. The Euler method is one of the simplest and earliest numerical methods for solving differential equations. This method is based on determining the next value of a function using the derivative at the previous point. The main advantage of the Euler method is its simplicity and clarity. Therefore, it is often used as an introductory method in the study of numerical techniques. However, the accuracy of this method is not very high. When the step size becomes large, the error also increases. For this reason, more accurate methods are preferred in complex calculations.

The Runge–Kutta method is one of the most popular numerical methods for solving differential equations. Compared to the Euler method, it provides higher accuracy and is widely used in practical computations. The essence of the Runge–Kutta method lies in calculating several derivative values within one step and using their average result. This helps reduce errors significantly.

The fourth-order Runge–Kutta method is the most commonly used version. It ensures high accuracy and computational efficiency. This method is widely applied in:

scientific calculations;
technical modeling;
physics and mechanics;
computer programming.

The Adams method is one of the multistep numerical methods. In this method, previous point values are used to determine the next value. The main advantage of the Adams method is its high computational speed because it does not require starting calculations from the beginning each time. Multistep methods provide effective results in large-scale computations and therefore occupy an important place in modern computational mathematics.

Numerical methods are widely applied in various fields of science and technology, including:

mechanics;
physics;
economics;
biology;
chemistry;
meteorology;
construction and engineering.

For example, in weather forecasting systems, complex mathematical models are calculated using numerical methods. Numerical techniques are also used in designing technical devices, analyzing electrical systems, and forecasting economic processes.

Numerical methods have several advantages:
They allow solving complex differential equations.
Calculations can be performed quickly using computers.
Results can be obtained even when analytical solutions do not exist.
They are effective in modeling practical problems.
They are widely used in scientific research.
However, numerical methods also have some disadvantages:
Results are approximate.

Computational errors may occur.

Accuracy decreases if the step size is chosen incorrectly.

Some methods require large computational effort.

Numerical methods play a significant role in modern computational mathematics because many real-life processes are described by differential equations whose exact analytical solutions cannot always be obtained. In such situations, numerical methods provide approximate solutions with sufficient accuracy for practical applications.

One of the important aspects of numerical methods is the concept of discretization. Discretization means dividing a continuous interval into a finite number of points. Instead of solving the equation continuously over the entire interval, the solution is calculated step by step at selected discrete points. This approach makes computer implementation possible and simplifies complex calculations.

Nevertheless, modern computer technologies significantly reduce these disadvantages. Solving differential equations using numerical methods is one of the important directions of computational mathematics. These methods make it possible to analyze complex mathematical models and solve practical problems. Euler, Runge–Kutta, and Adams methods are among the main techniques widely used for approximate solutions of differential equations. Each method has its own advantages and disadvantages, and they are selected depending on the type of problem. Today, numerical methods are applied in almost all fields of science and technology. With the development of computer technologies, the importance of these methods continues to grow. Therefore, studying and applying numerical methods is considered one of the important scientific tasks.

Accuracy shows how close the approximate solution is to the exact solution. Stability determines whether small computational errors grow during calculations or remain controlled. Computational complexity refers to the amount of time and resources required for performing calculations.

Another important concept in numerical methods is truncation error. Since numerical methods replace exact mathematical operations with approximate formulas, a certain level of error always appears. Reducing the step size generally decreases the truncation error, but it also increases the amount of computation required.

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